Mathematics

Task 1

Algebraic Expressions

1.1 Expanding brackets

Key points

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form $ax + b$, where $a \neq 0$ and $b \neq 0$, you create four terms. Two of these can usually be simplified by collecting like terms.

Example 1 Expand $4(3x - 2)$

$4(3x - 2) = 12x - 8$	Multiply everything inside the bracket by the 4 outside the bracket

Example 2 Expand and simplify $3(x + 5) - 4(2x + 3)$

$3(x + 5) - 4(2x + 3)$ $= 3x + 15 - 8x - 12$ $= 3 - 5x$	**1** Expand each set of brackets separately by multiplying $(x + 5)$ by 3 and $(2x + 3)$ by -4 **2** Simplify by collecting like terms: $3x - 8x = -5x$ and $15 - 12 = 3$

Example 3 Expand and simplify $(x + 3)(x + 2)$

$(x + 3)(x + 2)$ $= x(x + 2) + 3(x + 2)$ $= x^2 + 2x + 3x + 6$ $= x^2 + 5x + 6$	**1** Expand the brackets by multiplying $(x + 2)$ by x and $(x + 2)$ by 3 **2** Simplify by collecting like terms: $2x + 3x = 5x$

Example 4 Expand and simplify $(x - 5)(2x + 3)$

$(x - 5)(2x + 3)$ $= x(2x + 3) - 5(2x + 3)$	**1** Expand the brackets by multiplying $(2x + 3)$ by x and $(2x + 3)$ by -5

$= 2x^2 + 3x - 10x - 15$ $= 2x^2 - 7x - 15$	**2** Simplify by collecting like terms: $3x - 10x = -7x$

Practice

1 Expand.

a $3(2x - 1)$ **b** $-2(5pq + 4q^2)$ **c** $-(3xy - 2y^2)$

2 Expand and simplify.

a $7(3x + 5) + 6(2x - 8)$ **b** $8(5p - 2) - 3(4p + 9)$

c $9(3s + 1) - 5(6s - 10)$ **d** $2(4x - 3) - (3x + 5)$

3 Expand.

a $3x(4x + 8)$ **b** $4k(5k^2 - 12)$

c $-2h(6h^2 + 11h - 5)$ **d** $-3s(4s^2 - 7s + 2)$

4 Expand and simplify.

a $3(y^2 - 8) - 4(y^2 - 5)$ **b** $2x(x + 5) + 3x(x - 7)$

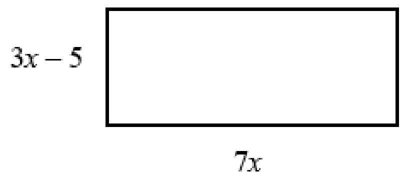
c $4p(2p - 1) - 3p(5p - 2)$ **d** $3b(4b - 3) - b(6b - 9)$

5 Expand $\frac{1}{2}(2y - 8)$

6 Expand and simplify.

a $13 - 2(m + 7)$ **b** $5p(p^2 + 6p) - 9p(2p - 3)$

7 The diagram shows a rectangle.
Write down an expression, in terms of x, for the area of the rectangle.
Show that the area of the rectangle can be written as $21x^2 - 35x$



8 Expand and simplify.

a $(x + 4)(x + 5)$ **b** $(x + 7)(x + 3)$ **c** $(x + 7)(x - 2)$

d $(x + 5)(x - 5)$ **e** $(2x + 3)(x - 1)$ **f** $(3x - 2)(2x + 1)$

g $(5x - 3)(2x - 5)$ **h** $(3x - 2)(7 + 4x)$ **i** $(3x + 4y)(5y + 6x)$

j $(x + 5)^2$ **k** $(2x - 7)^2$ **l** $(4x - 3y)^2$

Extend

9 Expand and simplify $(x + 3)^2 + (x - 4)^2$

10 Expand and simplify.

a $\left(x+\frac{1}{x}\right)\left(x-\frac{2}{x}\right)$ b $\left(x+\frac{1}{x}\right)^2$

1.2 Factorising Expressions

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation, find two numbers whose sum is b and whose product is ac.
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

Example 1 Factorise $15x^2y^3 + 9x^4y$

$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$	The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3 Factorise $x^2 + 3x - 10$

$b = 3$, $ac = -10$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and −2)
So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$	**2** Rewrite the b term ($3x$) using these two factors
$= x(x + 5) - 2(x + 5)$	**3** Factorise the first two terms and the last two terms
$= (x + 5)(x - 2)$	**4** $(x + 5)$ is a factor of both terms

Example 4 Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$ So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4) **2** Rewrite the b term ($-11x$) using these two factors **3** Factorise the first two terms and the last two terms **4** $(2x - 5)$ is a factor of both terms

Example 5 Simplify $\frac{x^2-4x-21}{2x^2+9x+9}$

$\frac{x^2-4x-21}{2x^2+9x+9}$	**1** Factorise the numerator and the denominator
For the numerator: $b = -4$, $ac = -21$	**2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (−7 and 3)
So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$	**3** Rewrite the b term ($-4x$) using these two factors **4** Factorise the first two terms and the last two terms
$= x(x - 7) + 3(x - 7)$	**5** $(x - 7)$ is a factor of both terms
$= (x - 7)(x + 3)$	
For the denominator: $b = 9$, $ac = 18$	**6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3)
So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$	**7** Rewrite the b term ($9x$) using these two factors **8** Factorise the first two terms and the last two terms
$= 2x(x + 3) + 3(x + 3)$	**9** $(x + 3)$ is a factor of both terms
$= (x + 3)(2x + 3)$	
So $\frac{x^2-4x-21}{2x^2+9x+9} = \frac{(x-7)(x+3)}{(x+3)(2x+3)}$ $= \frac{x-7}{2x+3}$	**10** $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

Practice

1 Factorise.

a $6x^4y^3 - 10x^3y^4$ **b** $21a^3b^5 + 35a^5b^2$ **c** $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

2 Factorise

a $x^2 + 7x + 12$ **b** $x^2 + 5x - 14$ **c** $x^2 - 11x + 30$

d $x^2 - 5x - 24$ **e** $x^2 - 7x - 18$ **f** $x^2 + x - 20$

g $x^2 - 3x - 40$ **h** $x^2 + 3x - 28$

3 Factorise

a $36x^2 - 49y^2$ **b** $4x^2 - 81y^2$ **c** $18a^2 - 200b^2c^2$

4 Factorise

a $2x^2 + x - 3$ **b** $6x^2 + 17x + 5$ **c** $2x^2 + 7x + 3$

d $9x^2 - 15x + 4$ **e** $10x^2 + 21x + 9$ **f** $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a $\frac{2x^2+4x}{x^2-x}$ **b** $\frac{x^2+3x}{x^2+2x-3}$ **c** $\frac{x^2-2x-8}{x^2-4x}$

d $\frac{x^2-5x}{x^2-25}$ **e** $\frac{x^2-x-12}{x^2-4x}$ **f** $\frac{2x^2+14x}{2x^2+4x-70}$

6 Simplify

a $\frac{9x^2-16}{3x^2+17x-28}$ **b** $\frac{2x^2-7x-15}{3x^2-17x+10}$

c $\frac{4-25x^2}{10x^2-11x-6}$ **d** $\frac{6x^2-x-1}{2x^2+7x-4}$

Extend

7 Simplify $\sqrt{x^2+10x+25}$

8 Simplify ¿¿

1.3 Laws of indices

Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the *n*th root of *a*
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1

Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $= 3^2$ $= 9$	**1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ **2** Use $\sqrt[3]{27} = 3$

Example 4 Evaluate 4^{-2}

$4^{-2} = \frac{1}{4^2}$	**1** Use the rule $a^{-m} = \frac{1}{a^m}$

$= \frac{1}{16}$	**2** Use $4^2 = 16$

Example 5 Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	6 ÷ 2 = 3 and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$

Example 6 Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	**1** Use the rule $a^m \times a^n = a^{m+n}$ **2** Use the rule $\frac{a^m}{a^n} = a^{m-n}$

Example 7 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged

Example 8 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}} \qquad = 4x^{\frac{-1}{2}}$	**1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ **2** Use the rule $\frac{1}{a^m} = a^{-m}$

Practice

1 Evaluate.

a 14^0 **b** 3^0 **c** 5^0 **d** x^0

2 Evaluate.

a $49^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $125^{\frac{1}{3}}$ **d** $16^{\frac{1}{4}}$

3 Evaluate.

a $25^{\frac{3}{2}}$ **b** $8^{\frac{5}{3}}$ **c** $49^{\frac{3}{2}}$ **d** $16^{\frac{3}{4}}$

4 Evaluate.

a 5^{-2} **b** 4^{-3} **c** 2^{-5} **d** 6^{-2}

5 Simplify.

a $\frac{3x^2 \times x^3}{2x^2}$ **b** $\frac{10x^5}{2x^2 \times x}$ **c** $\frac{3x \times 2x^3}{2x^3}$ **d** $\frac{7x^3 y^2}{14x^5 y}$

e $\frac{y^2}{y^{\frac{1}{2}} \times y}$ **f** $\frac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$ **g** $\frac{(2x^2)^3}{4x^0}$ **h** $\frac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

6 Evaluate.

a $4^{\frac{-1}{2}}$ **b** $27^{\frac{-2}{3}}$ **c** $9^{\frac{-1}{2}} \times 2^3$

d $16^{\frac{1}{4}} \times 2^{-3}$ **e** $\left(\frac{9}{16}\right)^{\frac{-1}{2}}$ **f** $\left(\frac{27}{64}\right)^{\frac{-2}{3}}$

7 Write the following as a single power of *x*.

a $\frac{1}{x}$ **b** $\frac{1}{x^7}$ **c** $\sqrt[4]{x}$

d $\sqrt[5]{x^2}$ **e** $\frac{1}{\sqrt[3]{x}}$ **f** $\frac{1}{\sqrt[3]{x^2}}$

8 Write the following without negative or fractional powers.

a x^{-3} **b** x^0 **c** $x^{\frac{1}{5}}$

d $x^{\frac{2}{5}}$ **e** $x^{\frac{-1}{2}}$ **f** $x^{\frac{-3}{4}}$

9 Write the following in the form ax^n.

a $5\sqrt{x}$ **b** $\frac{2}{x^3}$ **c** $\frac{1}{3x^4}$

d $\frac{2}{\sqrt{x}}$ **e** $\frac{4}{\sqrt[3]{x}}$ **f** 3

Extend

10 Write as sums of powers of *x*.

a $\frac{x^5+1}{x^2}$ **b** $x^2\left(x+\frac{1}{x}\right)$ **c** $x^{-4}\left(x^2+\frac{1}{x^3}\right)$

1.4 Surds

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5},$ etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$
- $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\frac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

Example 1 Simplify $\sqrt{50}$

$\sqrt{50}=\sqrt{25\times 2}$ ¿ $\sqrt{25}\times\sqrt{2}$ ¿ $5\times\sqrt{2}$ ¿ $5\sqrt{2}$	**1** Choose two numbers that are factors of 50. One of the factors must be a square number **2** Use the rule $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$ **3** Use $\sqrt{25}=5$

Example 2 Simplify $\sqrt{147}-2\sqrt{12}$

$\sqrt{147}-2\sqrt{12}$ ¿ $\sqrt{49\times 3}-2\sqrt{4\times 3}$ ¿ $\sqrt{49}\times\sqrt{3}-2\sqrt{4}\times\sqrt{3}$ ¿ $7\times\sqrt{3}-2\times 2\times\sqrt{3}$ ¿ $7\sqrt{3}-4\sqrt{3}$ ¿ $3\sqrt{3}$	**1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number **2** Use the rule $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$ **3** Use $\sqrt{49}=7$ and $\sqrt{4}=2$ **4** Collect like terms

Example 3 Simplify $(\sqrt{7}+\sqrt{2})(\sqrt{7}-\sqrt{2})$

$(\sqrt{7}+\sqrt{2})(\sqrt{7}-\sqrt{2})$ $= \sqrt{49}-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}-\sqrt{4}$ $= 7 - 2$ $= 5$	**1** Expand the brackets. A common mistake here is to write $(\sqrt{7})^2=49$ **2** Collect like terms: $-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}$ $=-\sqrt{7}\sqrt{2}+\sqrt{7}\sqrt{2}=0$

Example 4 Rationalise $\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{1 \times \sqrt{3}}{\sqrt{9}}$ $= \frac{\sqrt{3}}{3}$	**1** Multiply the numerator and denominator by $\sqrt{3}$ **2** Use $\sqrt{9}=3$

Example 5 Rationalise and simplify $\frac{\sqrt{2}}{\sqrt{12}}$

$\frac{\sqrt{2}}{\sqrt{12}} = \frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$ $= \frac{\sqrt{2} \times \sqrt{4 \times 3}}{12}$ $= \frac{2\sqrt{2}\sqrt{3}}{12}$	**1** Multiply the numerator and denominator by $\sqrt{12}$ **2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number **3** Use the rule $\sqrt{ab}=\sqrt{a} \times \sqrt{b}$ **4** Use $\sqrt{4}=2$ **5** Simplify the fraction: $\frac{2}{12}$ simplifies to $\frac{1}{6}$

$= \frac{\sqrt{2}\sqrt{3}}{6}$	

Example 6 Rationalise and simplify $\frac{3}{2+\sqrt{5}}$

$\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$	**1** Multiply the numerator and denominator by $2-\sqrt{5}$
$= \frac{3(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$	
$= \frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$	**2** Expand the brackets
$= \frac{6-3\sqrt{5}}{-1}$	**3** Simplify the fraction
$= 3\sqrt{5}-6$	**4** Divide the numerator by −1 Remember to change the sign of all terms when dividing by −1

Practice

1 Simplify.

a $\sqrt{45}$ **b** $\sqrt{125}$ **c** $\sqrt{48}$ **d** $\sqrt{175}$

e $\sqrt{300}$ **f** $\sqrt{28}$ **g** $\sqrt{72}$ **h** $\sqrt{162}$

2 Simplify.

a $\sqrt{72}+\sqrt{162}$ **b** $\sqrt{45}-2\sqrt{5}$ **c** $\sqrt{50}-\sqrt{8}$

d $\sqrt{75}-\sqrt{48}$ **e** $2\sqrt{28}+\sqrt{28}$ **f** $2\sqrt{12}-\sqrt{12}+\sqrt{27}$

3 Expand and simplify.

a $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$ **b** $(3+\sqrt{3})(5-\sqrt{12})$

c $(4-\sqrt{5})(\sqrt{45}+2)$ **d** $(5+\sqrt{2})(6-\sqrt{8})$

4 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{5}}$ **b** $\frac{1}{\sqrt{11}}$ **c** $\frac{2}{\sqrt{7}}$

d $\frac{2}{\sqrt{8}}$ **e** $\frac{2}{\sqrt{2}}$ **f** $\frac{5}{\sqrt{5}}$

g $\frac{\sqrt{8}}{\sqrt{24}}$ **h** $\frac{\sqrt{5}}{\sqrt{45}}$

5 Rationalise and simplify.

a $\frac{1}{3-\sqrt{5}}$ **b** $\frac{2}{4+\sqrt{3}}$ **c** $\frac{6}{5-\sqrt{2}}$

Extend

6 Expand and simplify $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})$

7 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{9}-\sqrt{8}}$ **b** $\frac{1}{\sqrt{x}-\sqrt{y}}$

Task 2 – Solving Quadratic Equations

2.1 Factorising

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation, find two numbers whose sum is *b* and whose products is *ac*.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$ $5x(x - 3) = 0$ So $5x = 0$ or $(x - 3) = 0$ Therefore $x = 0$ or $x = 3$	**1** Rearrange the equation so that all the terms are on one side of the equation and it is equal to zero. Do not divide both sides by *x* as this would lose the solution $x = 0$. **2** Factorise the quadratic equation. $5x$ is a common factor. **3** When two values multiply to make zero, at least one of the values must be zero. **4** Solve these two equations.

Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7$, $ac = 12$ $x^2 + 4x + 3x + 12 = 0$ $x(x + 4) + 3(x + 4) = 0$ $(x + 4)(x + 3) = 0$ So $(x + 4) = 0$ or $(x + 3) = 0$ Therefore $x = -4$ or $x = -3$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3) **2** Rewrite the *b* term ($7x$) using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x + 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$ So $(3x + 4) = 0$ or $(3x - 4) = 0$ $x = -\frac{4}{3}$ or $x = \frac{4}{3}$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$. **2** When two values multiply to make zero, at least one of the values must be zero. **3** Solve these two equations.

Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5$, $ac = -24$ So $2x^2 - 8x + 3x - 12 = 0$ $2x(x - 4) + 3(x - 4) = 0$ $(x - 4)(2x + 3) = 0$ So $(x - 4) = 0$ or $(2x + 3) = 0$ $x = 4$ or $x = -\frac{3}{2}$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (-8 and 3) **2** Rewrite the b term ($-5x$) using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x - 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Practice

1 Solve

a $6x^2 + 4x = 0$ **b** $28x^2 - 21x = 0$ **c** $x^2 + 7x + 10 = 0$

d $x^2 - 5x + 6 = 0$ **e** $x^2 - 3x - 4 = 0$ **f** $x^2 + 3x - 10 = 0$

g $x^2 - 10x + 24 = 0$ **h** $x^2 - 36 = 0$ **i** $x^2 + 3x - 28 = 0$

j $x^2 - 6x + 9 = 0$ **k** $2x^2 - 7x - 4 = 0$ **l** $3x^2 - 13x - 10 = 0$

2 Solve

a $x^2 - 3x = 10$ **b** $x^2 - 3 = 2x$ **c** $x^2 + 5x = 24$

d $x^2 - 42 = x$ **e** $x(x + 2) = 2x + 25$ **f** $x^2 - 30 = 3x - 2$

g $x(3x + 1) = x^2 + 15$ **h** $3x(x - 1) = 2(x + 1)$

2.2 Using the formula

Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
- If $b^2 - 4ac$ is negative, then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for *a*, *b* and *c*.

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1, b = 6, c = 4$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	**1** Identify *a*, *b* and *c* and write down the formula. Remember that $-b\pm\sqrt{b^2-4ac}$ is all over 2*a*, not just part of it.
$x=\frac{-6\pm\sqrt{6^2-4(1)(4)}}{2(1)}$	**2** Substitute $a = 1$, $b = 6$, $c = 4$ into the formula.
$x=\frac{-6\pm\sqrt{20}}{2}$	**3** Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.
$x=\frac{-6\pm2\sqrt{5}}{2}$	**4** Simplify $\sqrt{20}$. $\sqrt{20}=\sqrt{4\times5}=\sqrt{4}\times\sqrt{5}=2\sqrt{5}$
$x=-3\pm\sqrt{5}$	**5** Simplify by dividing numerator and denominator by 2.
So $x=-3-\sqrt{5}$ or $x=\sqrt{5}-3$	**6** Write down both the solutions.

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3, b = -7, c = -2$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	**1** Identify *a*, *b* and *c*, making sure you get the signs right and write down the formula. Remember that $-b\pm\sqrt{b^2-4ac}$ is all over 2*a*, not just part of it.
$x=-(-7)\pm\sqrt{¿¿¿}$	**2** Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.
$x=\frac{7\pm\sqrt{73}}{6}$	**3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.
So $x=\frac{7-\sqrt{73}}{6}$ or $x=\frac{7+\sqrt{73}}{6}$	**4** Write down both the solutions.

Practice

1 Solve, giving your solutions in surd form.

a $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

2 Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\frac{a \pm \sqrt{b}}{c}$, where a, b and c are integers.

3 Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

Extend

4 Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

a $4x(x - 1) = 3x - 2$

b $10 = (x + 1)^2$

c $x(3x - 1) = 10$

Task 3

Simultaneous Equations

3.1 Linear - Elimination

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$3x + y = 5$ $-\quad x + y = 1$ $2x \quad = 4$ So $x = 2$	**1** Subtract the second equation from the first equation to eliminate the y term.
Using $x + y = 1$ $2 + y = 1$ So $y = -1$	**2** To find the value of y, substitute $x = 2$ into one of the original equations.
Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$x + 2y = 13$ $+ \quad 5x - 2y = 5$ $6x = 18$ So $x = 3$	**1** Add the two equations together to eliminate the y term.
Using $x + 2y = 13$ $3 + 2y = 13$ So $y = 5$	**2** To find the value of y, substitute $x = 3$ into one of the original equations.
Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 3 Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$(2x + 3y = 2) \times 4 \rightarrow$ $8x + 12y = 8$ $(5x + 4y = 12) \times 3 \rightarrow$ $\underline{15x + 12y = 36}$ $7x = 28$ So $x = 4$	**1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term.
Using $2x + 3y = 2$ $2 \times 4 + 3y = 2$ So $y = -2$	**2** To find the value of y, substitute $x = 4$ into one of the original equations.
Check: equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $4x + y = 8$
$x + y = 5$

2 $3x + y = 7$
$3x + 2y = 5$

3 $4x + y = 3$
$3x - y = 11$

4 $3x + 4y = 7$
$x - 4y = 5$

5 $2x + y = 11$
$x - 3y = 9$

6 $2x + 3y = 11$
$3x + 2y = 4$

3.2 Linear – Substitution

Key points

- The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Example 1 Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

$5x + 3(2x + 1) = 14$ $5x + 6x + 3 = 14$ $11x + 3 = 14$	**1** Substitute $2x + 1$ for y into the second equation. **2** Expand the brackets and simplify.
$11x = 11$ So $x = 1$	**3** Work out the value of x.
Using $y = 2x + 1$ $y = 2 \times 1 + 1$ So $y = 3$	**4** To find the value of y, substitute $x = 1$ into one of the original equations.
Check: equation 1: $3 = 2 \times 1 + 1$ YES equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	**5** Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

$y = 2x - 16$ $4x + 3(2x - 16) = -3$	**1** Rearrange the first equation. **2** Substitute $2x - 16$ for y into the second equation. **3** Expand the brackets and simplify.
$4x + 6x - 48 = -3$ $10x - 48 = -3$ $10x = 45$	**4** Work out the value of x.
So $x = 4\frac{1}{2}$ Using $y = 2x - 16$ $y = 2 \times 4\frac{1}{2} - 16$ So $y = -7$	**5** To find the value of y, substitute $x = 4\frac{1}{2}$ into one of the original equations.
Check: equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES	**6** Substitute the values of x and y into both equations to check your answers.

Practice Solve these simultaneously equations.

1 $y = x - 4$
$2x + 5y = 43$

2 $y = 2x - 3$
$5x - 3y = 11$

3 $2y = 4x + 5$
$9x + 5y = 22$

4 $2x = y - 2$
$8x - 5y = -11$

5 $3x + 4y = 8$
$2x - y = -13$

6 $3y = 4x - 7$
$2y = 3x - 4$

7 $3x = y - 1$
$2y - 2x = 3$

8 $3x + 2y + 1 = 0$
$4y = 8 - x$

Extend

9 Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x+y)=\frac{3(y-x)}{4}$.

3.3 Linear and Quadratic

Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Example 1 Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$

$x^2 + (x + 1)^2 = 13$ $x^2 + x^2 + x + x + 1 = 13$ $2x^2 + 2x + 1 = 13$	**1** Substitute $x + 1$ for y into the second equation. **2** Expand the brackets and simplify.
$2x^2 + 2x - 12 = 0$ $(2x - 4)(x + 3) = 0$	**3** Factorise the quadratic equation.
So $x = 2$ or $x = -3$	**4** Work out the values of x.
Using $y = x + 1$ When $x = 2$, $y = 2 + 1 = 3$ When $x = -3$, $y = -3 + 1 = -2$ So the solutions are $x = 2$, $y = 3$ and $x = -3$, $y = -2$	**5** To find the value of y, substitute both values of x into one of the original equations.
Check: equation 1: $3 = 2 + 1$ YES and $-2 = -3 + 1$ YES	**6** Substitute both pairs of values of x and y into both equations to check your answers.

equation 2: $2^2 + 3^2 = 13$ YES and $(-3)^2 + (-2)^2 = 13$ YES	

Example 2 Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

$x=\frac{5-3y}{2}$	**1** Rearrange the first equation.
$2y^2+\left(\frac{5-3y}{2}\right)y=12$	**2** Substitute $\frac{5-3y}{2}$ for x into the second equation. Notice how it is easier to substitute for x than for y.
$2y^2+\frac{5y-3y^2}{2}=12$	**3** Expand the brackets and simplify.
$4y^2+5y-3y^2=24y^2+5y-24=0$	**4** Factorise the quadratic equation.
$(y + 8)(y - 3) = 0$ So $y = -8$ or $y = 3$	**5** Work out the values of y.
Using $2x + 3y = 5$ When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$ When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$	**6** To find the value of x, substitute both values of y into one of the original equations.
So the solutions are $x = 14.5$, $y = -8$ and $x = -2$, $y = 3$	**7** Substitute both pairs of values of x and y into both equations to check your answers.
Check: equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES and $2 \times (-2) + 3 \times 3 = 5$ YES equation 2: $2\times(-8)^2 + 14.5\times(-8) = 12$ YES and $2 \times (3)^2 + (-2) \times 3 = 12$ YES	

Practice

Solve these simultaneous equations.

1 $y = 2x + 1$ **2** $y = 6 - x$ **3** $y = x - 3$

$x^2 + y^2 = 10$

$x^2 + y^2 = 20$

$x^2 + y^2 = 5$

4 $y = 9 - 2x$
$x^2 + y^2 = 17$

5 $y = 3x - 5$
$y = x^2 - 2x + 1$

6 $y = x - 5$
$y = x^2 - 5x - 12$

7 $y = x + 5$
$x^2 + y^2 = 25$

8 $y = 2x - 1$
$x^2 + xy = 24$

9 $y = 2x$
$y^2 - xy = 8$

10 $2x + y = 11$
$xy = 15$

Extend

11 $x - y = 1$
$x^2 + y^2 = 3$

12 $y - x = 2$
$x^2 + xy = 3$

Task 4

Linear Inequalities

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

Example 1 Solve $-8 \le 4x < 16$

$-8 \le 4x < 16$ $-2 \le x < 4$	Divide all three terms by 4.

Example 2 Solve $4 \le 5x < 10$

$4 \le 5x < 10$ $\frac{4}{5} \le x < 2$	Divide all three terms by 5.

Example 3 Solve $2x - 5 < 7$

$2x - 5 < 7$ $2x < 12$ $x < 6$	**1** Add 5 to both sides. **2** Divide both sides by 2.

Example 4 Solve $2 - 5x \ge -8$

$2 - 5x \ge -8$ $-5x \ge -10$ $x \le 2$	**1** Subtract 2 from both sides. **2** Divide both sides by −5. Remember to reverse the inequality when dividing by a negative number.

Example 5 Solve $4(x - 2) > 3(9 - x)$

$4(x - 2) > 3(9 - x)$ $4x - 8 > 27 - 3x$ $7x - 8 > 27$	**1** Expand the brackets. **2** Add $3x$ to both sides. **3** Add 8 to both sides. **4** Divide both sides by 7.

$7x > 35$ $x > 5$	

Practice

1 Solve these inequalities.

a $4x > 16$ **b** $5x - 7 \leq 3$ **c** $1 \geq 3x + 4$

d $5 - 2x < 12$ **e** $\frac{x}{2} \geq 5$ **f** $8 < 3 - \frac{x}{3}$

2 Solve these inequalities.

a $\frac{x}{5} \leftarrow 4$ **b** $10 \geq 2x + 3$ **c** $7 - 3x > -5$

3 Solve

a $2 - 4x \geq 18$ **b** $3 \leq 7x + 10 < 45$ **c** $6 - 2x \geq 4$

d $4x + 17 < 2 - x$ **e** $4 - 5x < -3x$ **f** $-4x \geq 24$

4 Solve these inequalities.

a $3t + 1 < t + 6$ **b** $2(3n - 1) \geq n + 5$

5 Solve.

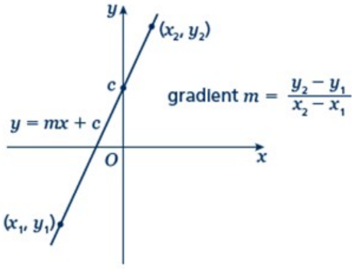
a $3(2 - x) > 2(4 - x) + 4$ **b** $5(4 - x) > 3(5 - x) + 2$

Extend

6 Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

Task 5

Straight Line Graphs



5.1 Equations

Key points

- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y-intercept
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a, b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

Example 1 A straight line has gradient $-\frac{1}{2}$ and y-intercept 3.
Write the equation of the line in the form $ax + by + c = 0$.

$m = -\frac{1}{2}$ and $c = 3$ So $y = -\frac{1}{2}x + 3$ $\frac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	**1** A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation. **2** Rearrange the equation so all the terms are on one side and 0 is on the other side. **3** Multiply both sides by 2 to eliminate the denominator.

Example 2 Find the gradient and the y-intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \frac{2}{3}x - \frac{4}{3}$ Gradient $= m = \frac{2}{3}$	**1** Make y the subject of the equation. **2** Divide all the terms by three to get the equation in the form $y = \ldots$ **3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

y-intercept = *c* = $-\frac{4}{3}$	

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$ $13 = 3 \times 5 + c$ $13 = 15 + c$ $c = -2$ $y = 3x - 2$	**1** Substitute the gradient given in the question into the equation of a straight line $y = mx + c$. **2** Substitute the coordinates $x = 5$ and $y = 13$ into the equation. **3** Simplify and solve the equation. **4** Substitute $c = -2$ into the equation $y = 3x + c$

Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2$, $x_2 = 8$, $y_1 = 4$ and $y_2 = 7$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{8-2} = \frac{3}{6} = \frac{1}{2}$	**1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$y = \frac{1}{2}x + c$ $4 = \frac{1}{2} \times 2 + c$ $c = 3$ $y = \frac{1}{2}x + 3$	**2** Substitute the gradient into the equation of a straight line $y = mx + c$. **3** Substitute the coordinates of either point into the equation. **4** Simplify and solve the equation. **5** Substitute $c = 3$ into the equation $y = \frac{1}{2}x + c$

Practice

1 Find the gradient and the *y*-intercept of the following equations.

a $y = 3x + 5$ **b** $y = -\frac{1}{2}x - 7$ **c** $2y = 4x - 3$

d $x + y = 5$ **e** $2x - 3y - 7 = 0$ **f** $5x + y - 4 = 0$

2 Find, in the form $ax + by + c = 0$ where a, b and c are integers, an equation for each of the lines with the following gradients and *y*-intercepts.

a gradient $\frac{-1}{2}$, *y*-intercept –7 **b** gradient 2, *y*-intercept 0

c gradient $\frac{2}{3}$, *y*-intercept 4 **d** gradient –1.2, *y*-intercept –2

3 Write an equation for the line which passes though the point (2, 5) and has gradient 4.

4 Write an equation for the line which passes through the point (6, 3) and has gradient $\frac{-2}{3}$

5 Write an equation for the line passing through each of the following pairs of points.

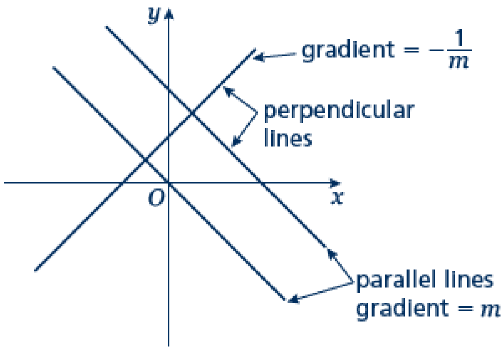
a (4, 5), (10, 17) **b** (0, 6), (–4, 8)

c (–1, –7), (5, 23) **d** (3, 10), (4, 7)

Extend

6 The equation of a line is $2y + 3x - 6 = 0$.
Write as much information as possible about this line.

5.2 Parallel and Perpendicular



Key points

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation $y = mx + c$ has gradient $\frac{-1}{m}$.

Example 1 Find the equation of the line parallel to $y = 2x + 4$ which passes through the point (4, 9).

$y = 2x + 4$ $m = 2$ $y = 2x + c$	**1** As the lines are parallel they have the same gradient. **2** Substitute $m = 2$ into the equation of a straight line $y = mx + c$.
$9 = 2 \times 4 + c$	**3** Substitute the coordinates into the equation $y = 2x + c$
$9 = 8 + c$ $c = 1$	**4** Simplify and solve the equation.
$y = 2x + 1$	**5** Substitute $c = 1$ into the equation $y = 2x + c$

Example 2 Find the equation of the line perpendicular to $y = 2x - 3$ which passes through the point (−2, 5).

$y = 2x - 3$ $m = 2$ $-\frac{1}{m} = -\frac{1}{2}$	**1** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.
$y = -\frac{1}{2}x + c$	**2** Substitute $m = -\frac{1}{2}$ into $y = mx + c$.
$5 = -\frac{1}{2} \times (-2) + c$	**3** Substitute the coordinates (–2, 5) into the equation $y = -\frac{1}{2}x + c$

$5 = 1 + c$ $c = 4$ $y = -\frac{1}{2}x + 4$	**4** Simplify and solve the equation. **5** Substitute $c = 4$ into $y = \frac{-1}{2}x + c$.

Example 3 A line passes through the points (0, 5) and (9, −1).
Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$x_1 = 0$, $x_2 = 9$, $y_1 = 5$ and $y_2 = -1$	**1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{9-0} = \frac{-6}{9} = \frac{-2}{3}$ $\frac{-1}{m} = \frac{3}{2}$	**2** As the lines are perpendicular, the gradient of the perpendicular line is $\frac{-1}{m}$.
$y = \frac{3}{2}x + c$	**3** Substitute the gradient into the equation $y = mx + c$.
Midpoint = $\left(\frac{0+9}{2}, \frac{5+(-1)}{2}\right) = \left(\frac{9}{2}, 2\right)$	**4** Work out the coordinates of the midpoint of the line.
$2 = \frac{3}{2} \times \frac{9}{2} + c$	**5** Substitute the coordinates of the midpoint into the equation.
$c = \frac{-19}{4}$	**6** Simplify and solve the equation.
$y = \frac{3}{2}x - \frac{19}{4}$	**7** Substitute $c = -\frac{19}{4}$ into the equation $y = \frac{3}{2}x + c$.

Practice

1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

a $y = 3x + 1$ (3, 2) **b** $y = 3 - 2x$ (1, 3)

c $2x + 4y + 3 = 0$ (6, –3) **d** $2y - 3x + 2 = 0$ (8, 20)

2 Find the equation of the line perpendicular to $y = \frac{1}{2}x - 3$ which passes through the point (–5, 3).

3 Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

a $y = 2x - 6$ (4, 0) **b** $y = \frac{-1}{3}x + \frac{1}{2}$ (2, 13)

c $x - 4y - 4 = 0$ (5, 15) **d** $5y + 2x - 5 = 0$ (6, 7)

4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

a (4, 3), (–2, –9) **b** (0, 3), (–10, 8)

Extend

5 Work out whether these pairs of lines are parallel, perpendicular or neither.

a $y = 2x + 3$, $y = 2x - 7$

b $y = 3x$, $2x + y - 3 = 0$

c $y = 4x - 3$, $4y + x = 2$

d $3x - y + 5 = 0$, $x + 3y = 1$

e $2x + 5y - 1 = 0$, $y = 2x + 7$

f $2x - y = 6$, $6x - 3y + 3 = 0$

6 The straight line $\mathbf{L_1}$ passes through the points *A* and *B* with coordinates (–4, 4) and (2, 1), respectively.

a Find the equation of $\mathbf{L_1}$ in the form $ax + by + c = 0$

The line $\mathbf{L_2}$ is parallel to the line $\mathbf{L_1}$ and passes through the point *C* with coordinates (–8, 3).

b Find the equation of $\mathbf{L_2}$ in the form $ax + by + c = 0$

The line $\mathbf{L_3}$ is perpendicular to the line $\mathbf{L_1}$ and passes through the origin.

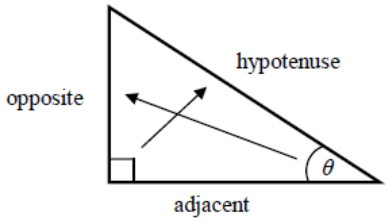
c Find an equation of $\mathbf{L_3}$

Task 6

Trigonometry

6.1 Right-angled

Key points



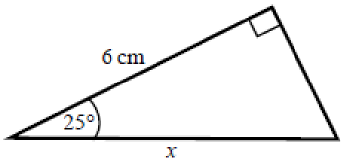
- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle θ is called the opposite
  - the side next to the angle θ is called the adjacent.

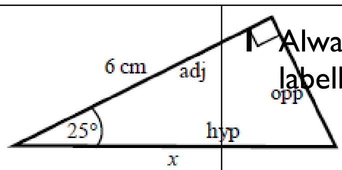
- In a right-angled triangle:
  - the ratio of the opposite side to the hypotenuse is the sine of angle θ, $\sin\theta = \frac{\text{opp}}{\text{hyp}}$
  - the ratio of the adjacent side to the hypotenuse is the cosine of angle θ, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$
  - the ratio of the opposite side to the adjacent side is the tangent of angle θ, $\tan\theta = \frac{\text{opp}}{\text{adj}}$

- The sine, cosine and tangent of some angles may be written exactly.

	0	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

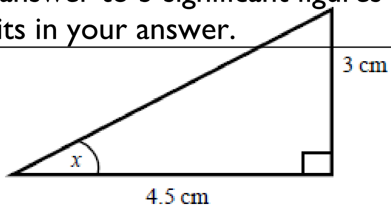
Example 1 Calculate the length of side *x*, correct to 3 significant figures.

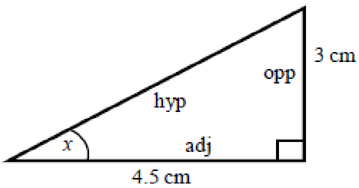


	**1** Always start by labelling the sides.
$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	**2** You are given the adjacent and the hypotenuse so use the cosine ratio.
$\cos 25° = \frac{6}{x}$ $x = \frac{6}{\cos 25°}$	**3** Substitute the sides and angle into the cosine ratio.

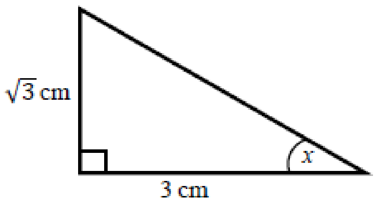
$x = 6.620\,267\,5...$ $x = 6.62$ cm	**4** Rearrange to make *x* the subject. **5** Use your calculator to work out $6 \div \cos 25°$. **6** Round your answer to 3 significant figures and write the units in your answer.

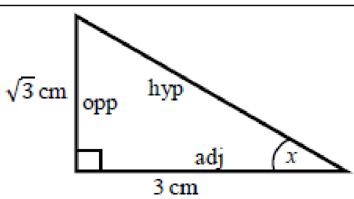
Example 2 Calculate the size of angle *x*.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the sides.
$\tan\theta = \frac{\text{opp}}{\text{adj}}$	**2** You are given the opposite and the adjacent so use the tangent ratio.
$\tan x = \frac{3}{4.5}$	**3** Substitute the sides and angle into the tangent ratio.
$x = \tan^{-1}\left(\frac{3}{4.5}\right)$	**4** Use $\tan^{-1}$ to find the angle.
$x = 33.690\,067\,5...$	**5** Use your calculator to work out $\tan^{-1}(3 \div 4.5)$.
$x = 33.7°$	**6** Round your answer to 3 significant figures and write the units in your answer.

Example 3 Calculate the exact size of angle *x*.



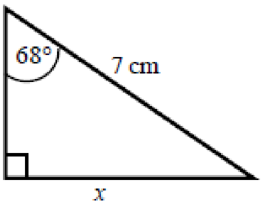
	**1** Always start by labelling the sides. **2** You are given the opposite and the adjacent so use the tangent ratio.

$\tan\theta = \frac{\text{opp}}{\text{adj}}$ $\tan x = \frac{\sqrt{3}}{3}$ $x = 30°$	**3** Substitute the sides and angle into the tangent ratio. **4** Use the table from the key points to find the angle.

Practice

1 Calculate the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

a



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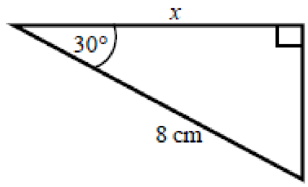

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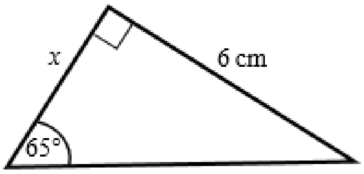

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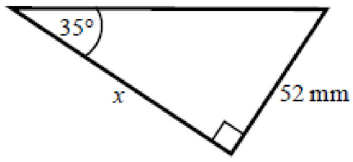

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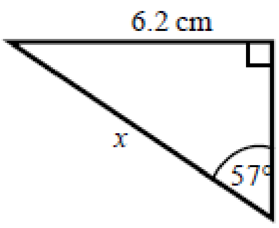

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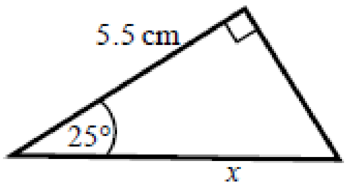
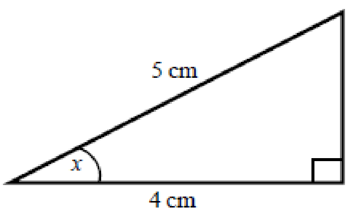


2 Calculate the size of angle *x* in each triangle.
Give your answers correct to 1 decimal place.

a



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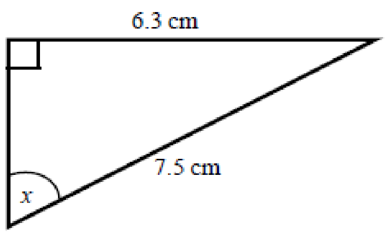

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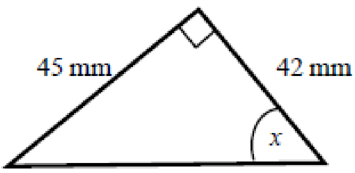

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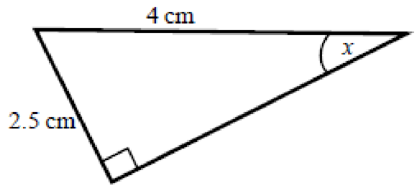
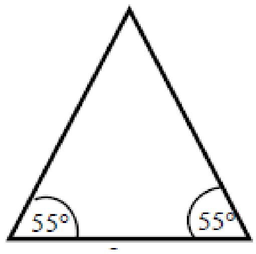
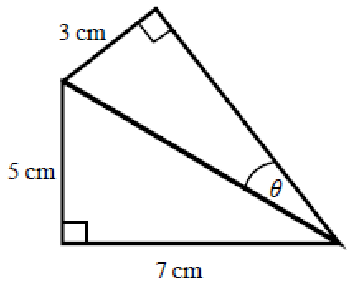


3 Work out the height of the isosceles triangle.
Give your answer correct to 3 significant figures.

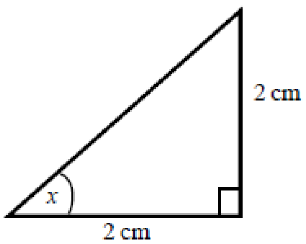


4 Calculate the size of angle θ.
Give your answer correct to 1 decimal place.



5 Find the exact value of *x* in each triangle.

a



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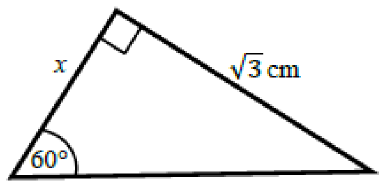

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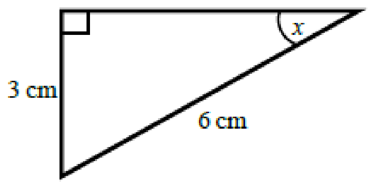

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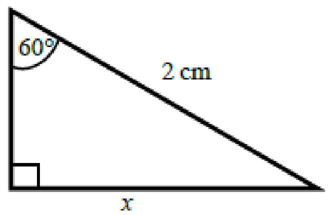
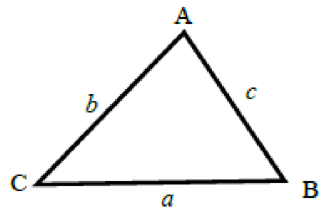


6.2 Cosine Rule

Key points

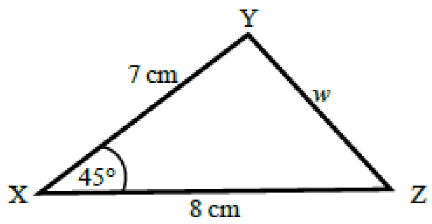
- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

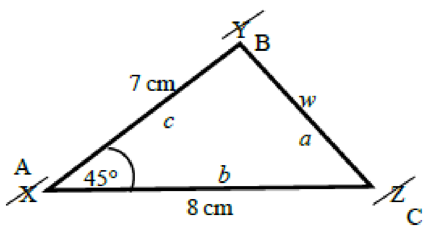


- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc\cos A$.

- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

Example 4 Work out the length of side *w*.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the angles and sides. **2** Write the cosine rule to find the side. **3** Substitute the values *a*, *b* and *A* into the formula. **4** Use a calculator to find w^2 and then *w*. **5** Round your final answer to 3 significant figures and write the units in your answer.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

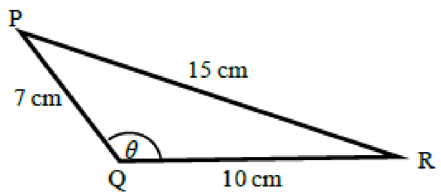
$$w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45°$$

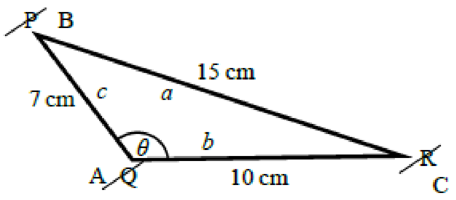
w^2 = 33.804 040 51...

$w = \sqrt{33.80404051}$

w = 5.81 cm

Example 5 Work out the size of angle θ.
Give your answer correct to 1 decimal place.





1 Always start by labelling the angles and sides.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos\theta = \frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$$

$$\cos\theta = \frac{-76}{140}$$

θ = 122.878 349...

θ = 122.9°

2 Write the cosine rule to find the angle.

3 Substitute the values *a*, *b* and *c* into the formula.

4 Use $\cos^{-1}$ to find the angle.

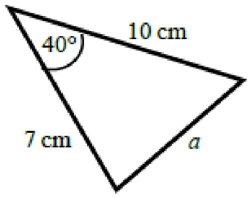
5 Use your calculator to work out $\cos^{-1}(-76 \div 140)$.

6 Round your answer to 1 decimal place and write the units in your answer.

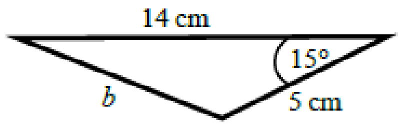
Practice

1 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

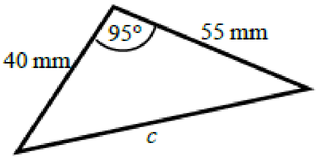
a



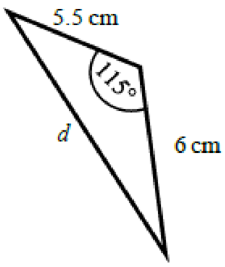
b



c

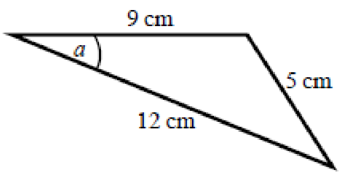


d

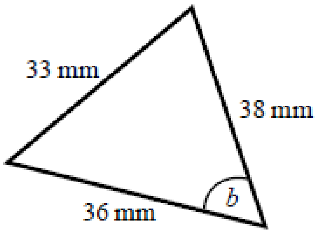


2 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

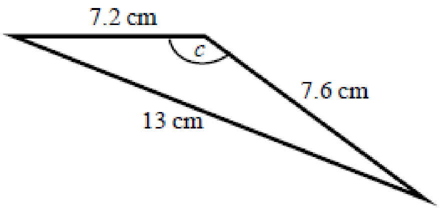
a



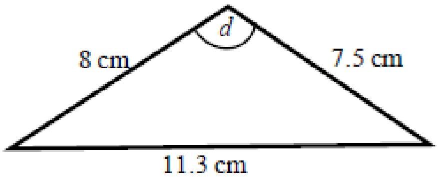
b



c

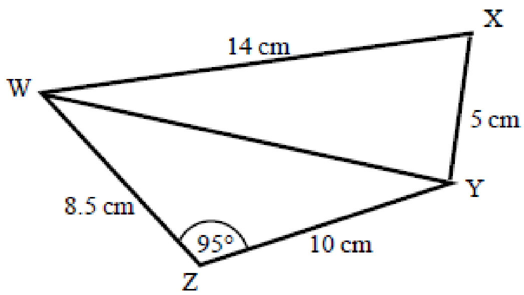


d



3 **a** Work out the length of WY.
Give your answer correct to
3 significant figures.

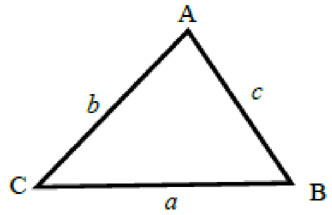
b Work out the size of angle WXY.
Give your answer correct to
1 decimal place.



6.3 Sine Rule

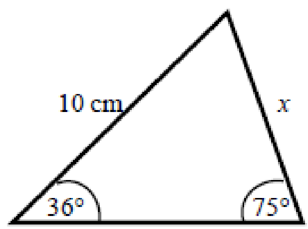
Key points

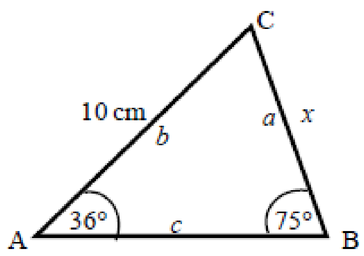
- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.



- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$.
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$.

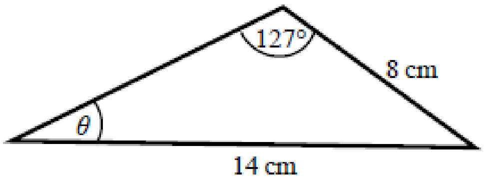
Example 6 Work out the length of side *x*.
Give your answer correct to 3 significant figures.

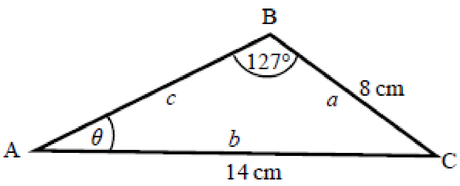


	**1** Always start by labelling the angles and sides.
$\frac{a}{\sin A}=\frac{b}{\sin B}$	**2** Write the sine rule to find the side.
$\frac{x}{\sin 36°}=\frac{10}{\sin 75°}$	**3** Substitute the values *a*, *b*, *A* and *B* into the formula.
$x=\frac{10\times\sin 36°}{\sin 75°}$	**4** Rearrange to make *x* the subject.

x = 6.09 cm	**5** Round your answer to 3 significant figures and write the units in your answer.

Example 7 Work out the size of angle θ.
Give your answer correct to 1 decimal place.

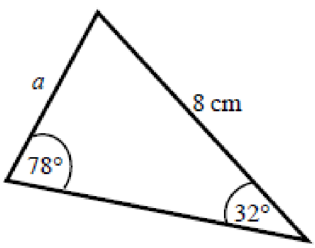


	**1** Always start by labelling the angles and sides.
$\frac{\sin A}{a} = \frac{\sin B}{b}$	**2** Write the sine rule to find the angle.
$\frac{\sin \theta}{8} = \frac{\sin 127°}{14}$	**3** Substitute the values a, b, A and B into the formula.
$\sin \theta = \frac{8 \times \sin 127°}{14}$	**4** Rearrange to make sin θ the subject.
θ = 27.2°	**5** Use $\sin^{-1}$ to find the angle. Round your answer to 1 decimal place and write the units in your answer.

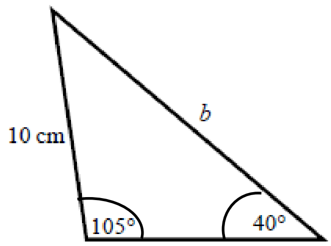
Practice

1 Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

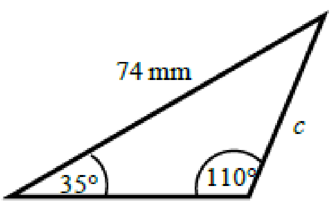
a



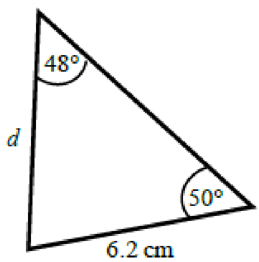
b



c

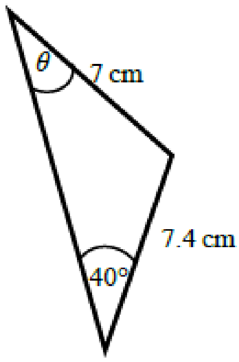


d

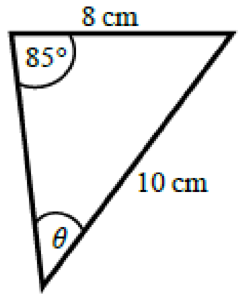


2 Calculate the angles labelled ϑ in each triangle.
Give your answer correct to 1 decimal place.

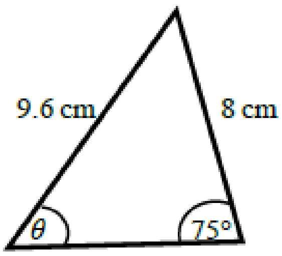
a



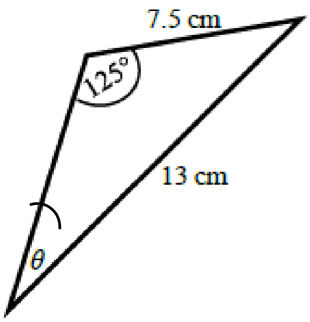
b



c

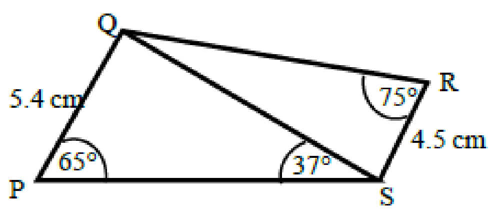


d



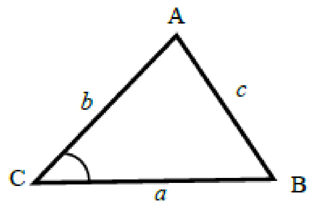
3 **a** Work out the length of QS.
Give your answer correct to 3 significant figures.

b Work out the size of angle RQS.
Give your answer correct to 1 decimal place.



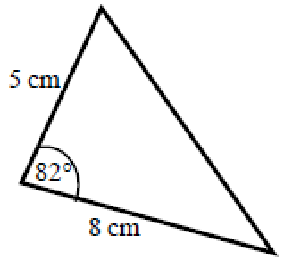
6.4 Area of a triangle

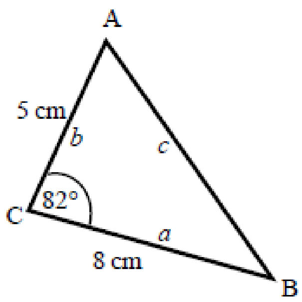
Key points



- *a* is the side opposite angle A. *b* is the side opposite angle B. *c* is the side opposite angle C.
- The area of the triangle is $\frac{1}{2}ab\sin C$.

Example 8 Find the area of the triangle.

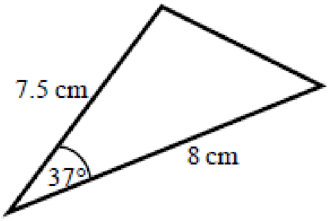


	**1** Always start by labelling the sides and angles of the triangle.
Area = $\frac{1}{2}ab\sin C$ Area = $\frac{1}{2}\times 8\times 5\times \sin 82°$ Area = 19.805 361... Area = 19.8 cm^2	**2** State the formula for the area of a triangle. **3** Substitute the values of *a*, *b* and *C* into the formula for the area of a triangle. **4** Use a calculator to find the area. **5** Round your answer to 3 significant figures and write the units in your answer.

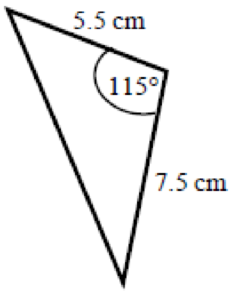
Practice

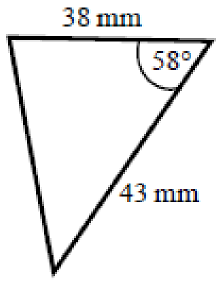
1 Work out the area of each triangle.
Give your answers correct to 3 significant figures.

a

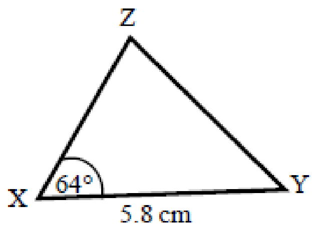


b **c**





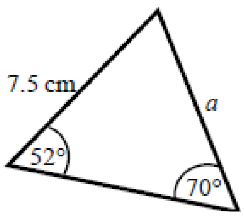
2 The area of triangle XYZ is 13.3 cm^2.
Work out the length of XZ.



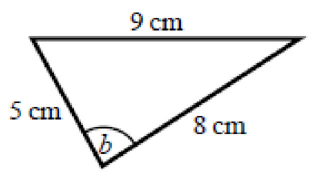
Extend

3 Find the size of each lettered angle or side.
Give your answers correct to 3 significant figures.

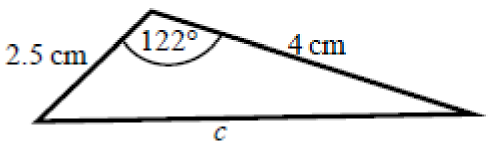
a



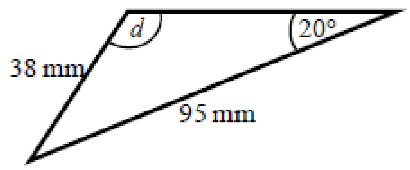
b



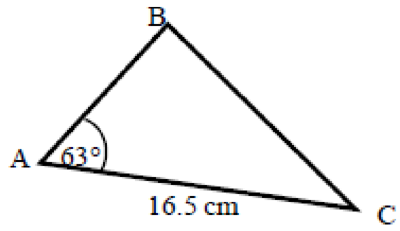
c



d



4 The area of triangle ABC is 86.7 cm^2.
Work out the length of BC.
Give your answer correct to 3 significant figures.



Answers

Task 1

Algebraic Expressions

1.1 Expanding brackets

1 **a** $6x - 3$ **b** $-10pq - 8q^2$ **c** $-3xy + 2y^2$

2 **a** $21x + 35 + 12x - 48 = 33x - 13$ **b** $40p - 16 - 12p - 27 = 28p - 43$
c $27s + 9 - 30s + 50 = -3s + 59 = 59 - 3s$ **d** $8x - 6 - 3x - 5 = 5x - 11$

3 **a** $12x^2 + 24x$ **b** $20k^3 - 48k$
c $10h - 12h^3 - 22h^2$ **d** $21s^2 - 21s^3 - 6s$

4 **a** $-y^2 - 4$ **b** $5x^2 - 11x$
c $2p - 7p^2$ **d** $6b^2$

5 $y - 4$

6 **a** $-1 - 2m$ **b** $5p^3 + 12p^2 + 27p$

7 $7x(3x - 5) = 21x^2 - 35x$

8 **a** $x^2 + 9x + 20$ **b** $x^2 + 10x + 21$
c $x^2 + 5x - 14$ **d** $x^2 - 25$
e $2x^2 + x - 3$ **f** $6x^2 - x - 2$
g $10x^2 - 31x + 15$ **h** $12x^2 + 13x - 14$
i $18x^2 + 39xy + 20y^2$ **j** $x^2 + 10x + 25$
k $4x^2 - 28x + 49$ **l** $16x^2 - 24xy + 9y^2$

9 $2x^2 - 2x + 25$

10 **a** $x^2 - 1 - \frac{2}{x^2}$ **b** $x^2 + 2 + \frac{1}{x^2}$

1.2 Factorising Expressions

1 **a** $2x^3y^3(3x - 5y)$ **b** $7a^3b^2(3b^3 + 5a^2)$ **c** $5x^2y^2(5 - 2x + 3y)$

2 **a** $(x + 3)(x + 4)$ **b** $(x + 7)(x - 2)$ **c** $(x - 5)(x - 6)$

d $(x-8)(x+3)$ **e** $(x-9)(x+2)$ **f** $(x+5)(x-4)$
g $(x-8)(x+5)$ **h** $(x+7)(x-4)$

3 **a** $(6x-7y)(6x+7y)$ **b** $(2x-9y)(2x+9y)$ **c** $2(3a-10bc)(3a+10bc)$

4 **a** $(x-1)(2x+3)$ **b** $(3x+1)(2x+5)$ **c** $(2x+1)(x+3)$

d $(3x-1)(3x-4)$ **e** $(5x+3)(2x+3)$ **f** $2(3x-2)(2x-5)$

5 **a** $\frac{2(x+2)}{x-1}$ **b** $\frac{x}{x-1}$ **c** $\frac{x+2}{x}$

d $\frac{x}{x+5}$ **e** $\frac{x+3}{x}$ **f** $\frac{x}{x-5}$

6 **a** $\frac{3x+4}{x+7}$ **b** $\frac{2x+3}{3x-2}$
c $\frac{2-5x}{2x-3}$ **d** $\frac{3x+1}{x+4}$

7 $(x+5)$

8 $\frac{4(x+2)}{x-2}$

1.3 Laws of indices

1 **a** 1 **b** 1 **c** 1 **d** 1

2 **a** 7 **b** 4 **c** 5 **d** 2

3 **a** 125 **b** 32 **c** 343 **d** 8

4 **a** $\frac{1}{25}$ **b** $\frac{1}{64}$ **c** $\frac{1}{32}$ **d** $\frac{1}{36}$

5 **a** $\frac{3x^3}{2}$ **b** $5x^2$ **c** $3x$ **d** $\frac{y}{2x^2}$

e $y^{\frac{1}{2}}$ **f** c^{-3} **g** $2x^6$ **h** x

6	**a**	$\frac{1}{2}$	**b**	$\frac{1}{9}$	**c**	$\frac{8}{3}$
	d	$\frac{1}{4}$	**e**	$\frac{4}{3}$	**f**	$\frac{16}{9}$
7	**a**	x^{-1}	**b**	x^{-7}	**c**	$x^{\frac{1}{4}}$
	d	$x^{\frac{2}{5}}$	**e**	$x^{-\frac{1}{3}}$	**f**	$x^{-\frac{2}{3}}$
8	**a**	$\frac{1}{x^3}$	**b**	1	**c**	$\sqrt[5]{x}$
	d	$\sqrt[5]{x^2}$	**e**	$\frac{1}{\sqrt{x}}$	**f**	$\frac{1}{\sqrt[4]{x^3}}$
9	**a**	$5x^{\frac{1}{2}}$	**b**	$2x^{-3}$	**c**	$\frac{1}{3}x^{-4}$
	d	$2x^{-\frac{1}{2}}$	**e**	$4x^{-\frac{1}{3}}$	**f**	$3x^0$
10	**a**	$x^3 + x^{-2}$	**b**	$x^3 + x$	**c**	$x^{-2} + x^{-7}$

1.4 Surds

1	**a**	$3\sqrt{5}$	**b**	$5\sqrt{5}$	**c**	$4\sqrt{3}$	**d**	$5\sqrt{7}$
	e	$10\sqrt{3}$	**f**	$2\sqrt{7}$	**g**	$6\sqrt{2}$	**h**	$9\sqrt{2}$
2	**a**	$15\sqrt{2}$	**b**	$\sqrt{5}$	**c**	$3\sqrt{2}$	**d**	$\sqrt{3}$
	e	$6\sqrt{7}$	**f**	$5\sqrt{3}$				
3	**a**	−1	**b**	$9-\sqrt{3}$	**c**	$10\sqrt{5}-7$	**d**	$26-4\sqrt{2}$
4	**a**	$\frac{\sqrt{5}}{5}$	**b**	$\frac{\sqrt{11}}{11}$	**c**	$\frac{2\sqrt{7}}{7}$	**d**	$\frac{\sqrt{2}}{2}$
	e	$\sqrt{2}$	**f**	$\sqrt{5}$	**g**	$\frac{\sqrt{3}}{3}$	**h**	$\frac{1}{3}$

5 **a** $\frac{3+\sqrt{5}}{4}$ **b** $\frac{2(4-\sqrt{3})}{13}$ **c** $\frac{6(5+\sqrt{2})}{23}$

6 $x - y$

7 **a** $3+2\sqrt{2}$ **b** $\frac{\sqrt{x}+\sqrt{y}}{x-y}$

Task 2

Solving Quadratic Equations

2.1 Factorising

1 **a** $x = 0$ or $x = -\frac{2}{3}$ **b** $x = 0$ or $x = \frac{3}{4}$
c $x = -5$ or $x = -2$ **d** $x = 2$ or $x = 3$
e $x = -1$ or $x = 4$ **f** $x = -5$ or $x = 2$
g $x = 4$ or $x = 6$ **h** $x = -6$ or $x = 6$
i $x = -7$ or $x = 4$ **j** $x = 3$
k $x = -\frac{1}{2}$ or $x = 4$ **l** $x = -\frac{2}{3}$ or $x = 5$

2 **a** $x = -2$ or $x = 5$ **b** $x = -1$ or $x = 3$
c $x = -8$ or $x = 3$ **d** $x = -6$ or $x = 7$
e $x = -5$ or $x = 5$ **f** $x = -4$ or $x = 7$
g $x = -3$ or $x = 2\frac{1}{2}$ **h** $x = -\frac{1}{3}$ or $x = 2$

2.2 Using the Formula

1 **a** $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$ **b** $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

2 $x = \frac{7+\sqrt{41}}{2}$ or $x = \frac{7-\sqrt{41}}{2}$

3 $x = \frac{-3+\sqrt{89}}{20}$ or $x = \frac{-3-\sqrt{89}}{20}$

4 **a** $x = \frac{7+\sqrt{17}}{8}$ or $x = \frac{7-\sqrt{17}}{8}$

b $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$

c $x = -1\frac{2}{3}$ or $x = 2$

Task 3

Simultaneous Equations

3.1 Linear - Elimination

1 $x = 1, y = 4$

2 $x = 3, y = -2$

3 $x = 2, y = -5$

4 $x = 3, y = -\frac{1}{2}$

5 $x = 6, y = -1$

6 $x = -2, y = 5$

3.2 Linear - Substitution

1 $x = 9, y = 5$

2 $x = -2, y = -7$

3 $x = \frac{1}{2}, y = 3\frac{1}{2}$

4 $x = \frac{1}{2}, y = 3$

5 $x = -4, y = 5$

6 $x = -2, y = -5$

7 $x = \frac{1}{4}, y = 1\frac{3}{4}$

8 $x = -2, y = 2\frac{1}{2}$

9 $x = -2\frac{1}{2}, y = 5\frac{1}{2}$

3.3 Linear and Quadratic

1 $x = 1, y = 3$
$x = -\frac{9}{5}, y = -\frac{13}{5}$

2 $x = 2, y = 4$
$x = 4, y = 2$

3 $x = 1, y = -2$
$x = 2, y = -1$

4 $x = 4, y = 1$
$x = \frac{16}{5}, y = \frac{13}{5}$

5 $x = 3, y = 4$
$x = 2, y = 1$

6 $x = 7, y = 2$
$x = -1, y = -6$

7 $x = 0, y = 5$
$x = -5, y = 0$

8 $x = -\frac{8}{3}, y = -\frac{19}{3}$
$x = 3, y = 5$

9 $x = -2, y = -4$
$x = 2, y = 4$

10 $x = \frac{5}{2}, y = 6$
$x = 3, y = 5$

11 $x = \frac{1+\sqrt{5}}{2}, y = \frac{-1+\sqrt{5}}{2}$
$x = \frac{1-\sqrt{5}}{2}, y = \frac{-1-\sqrt{5}}{2}$

12 $x = \frac{-1+\sqrt{7}}{2}$, $y = \frac{3+\sqrt{7}}{2}$

$x = \frac{-1-\sqrt{7}}{2}$, $y = \frac{3-\sqrt{7}}{2}$

Task 4

Linear Inequalities

1 **a** $x > 4$ **b** $x \le 2$ **c** $x \le -1$

d $x > -\frac{7}{2}$ **e** $x \ge 10$ **f** $x < -15$

2 **a** $x < -20$ **b** $x \le 3.5$ **c** $x < 4$

3 **a** $x \le -4$ **b** $-1 \le x < 5$ **c** $x \le 1$

d $x < -3$ **e** $x > 2$ **f** $x \le -6$

4 **a** $t < \frac{5}{2}$ **b** $n \ge \frac{7}{5}$

5 **a** $x < -6$ **b** $x < \frac{3}{2}$

6 $x > 5$ (which also satisfies $x > 3$)

Task 5

Straight Line Graphs

5.1 Equations

1 **a** $m = 3, c = 5$ **b** $m = -\frac{1}{2}, c = -7$

c $m = 2, c = -\frac{3}{2}$ **d** $m = -1, c = 5$

e $m = \frac{2}{3}, c = -\frac{7}{3}$ or $-2\frac{1}{3}$ **f** $m = -5, c = 4$

2 **a** $x + 2y + 14 = 0$ **b** $2x - y = 0$ **c** $2x - 3y + 12 = 0$ **d** $6x + 5y + 10 = 0$

3 $y = 4x - 3$

4 $y = -\frac{2}{3}x + 7$

5 **a** $y = 2x - 3$ **b** $y = -\frac{1}{2}x + 6$ **c** $y = 5x - 2$ **d** $y = -3x + 19$

6 $y = -\frac{3}{2}x + 3$, the gradient is $-\frac{3}{2}$ and the y-intercept is 3.
The line intercepts the axes at (0, 3) and (2, 0).

Students may sketch the line or give coordinates that lie on the line such as $\left(1, \frac{3}{2}\right)$ or $(4, -3)$.

5.2 Parallel and Perpendicular

1 **a** $y = 3x - 7$ **b** $y = -2x + 5$ **c** $y = -\frac{1}{2}x$ **d** $y = \frac{3}{2}x + 8$

2 $y = -2x - 7$

3 **a** $y = -\frac{1}{2}x + 2$ **b** $y = 3x + 7$ **c** $y = -4x + 35$ **d** $y = \frac{5}{2}x - 8$

4 **a** $y = -\frac{1}{2}x$ **b** $y = 2x$

5	**a**	Parallel	**b**	Neither	**c**	Perpendicular		
	d	Perpendicular	**e**	Neither	**f**	Parallel		
6	**a**	$x + 2y - 4 = 0$	**b**	$x + 2y + 2 = 0$	**c**	$y = 2x$		

Task 6

Trigonometry

6.1 Right-angled

1	**a**	6.49 cm	**b**	6.93 cm	**c**	2.80 cm		
	d	74.3 mm	**e**	7.39 cm	**f**	6.07 cm		
2	**a**	36.9°	**b**	57.1°	**c**	47.0°	**d**	38.7°
3	5.71 cm							
4	20.4°							
5	**a**	45°	**b**	1 cm	**c**	30°	**d**	$\sqrt{3}$ cm

6.2 Cosine Rule

1	**a**	6.46 cm	**b**	9.26 cm	**c**	70.8 mm	**d**	9.70 cm
2	**a**	22.2°	**b**	52.9°	**c**	122.9°	**d**	93.6°
3	**a**	13.7 cm	**b**	76.0°				

6.3 Sine Rule

1	**a**	4.33 cm	**b**	15.0 cm	**c**	45.2 mm	**d**	6.39 cm
2	**a**	42.8°	**b**	52.8°	**c**	53.6°	**d**	28.2°
3	**a**	8.13 cm	**b**	32.3°				

6.4 Area of a triangle

1	**a**	18.1 cm^2	**b**	18.7 cm^2	**c**	693 mm^2		
2	5.10 cm							
3	**a**	6.29 cm	**b**	84.3°	**c**	5.73 cm	**d**	58.8°
4	15.3 cm							