Year 12 Maths A-level Induction work

Introduction

Thank you for choosing to study Mathematics in the sixth form at Nailsea School.

Over the course, you will study topics in Pure Maths, Mechanics and Statistics.

The Mathematics Department is committed to ensuring that you make good progress throughout your A level course. In order that you make the best possible start to the course, we have prepared this booklet. It is vital that you spend time working through the questions in this booklet over the summer. You need to have a good knowledge of these topics before you commence your course in September. You should have met all the topics before at GCSE.

Work through what you need to from each chapter, making sure that you understand the examples. Then tackle the exercise to ensure you understand the topic thoroughly. The answers are at the back of the booklet. You will need to be organised so keep your work in a folder and mark any queries to ask at the beginning of term.

In addition to the work in this booklet you can also use the following to help with your studies:

- Hegarty Maths
  - At the back of the booklet is a table outlining all the skills that you need to master for A level maths, with clip numbers.
- Hegarty Maths Live Maths Lessons "Getting ready for A-level Maths"
  - Videos on content that is essential for A level maths, these are different to the videos on the Hegarty Maths website. Search "Hegarty Maths a level" on YouTube.
- Alevelmathsrevision.com/bridging-the-gap/
  - Tutorial videos, questions and solutions on essential content can be found on here.

In the first or second week of term you will take a test to check how well you understand these topics, so it is important that you have completed the booklet by then.

Use this introduction to give you a good start to your Year 12 work that will help you to enjoy, and benefit from, the course. The more effort you put in, right from the start, the better you will do.

Contents

1 Algebraic Expressions

1.1 Expanding brackets

Key points

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form $ax + b$, where $a \neq 0$ and $b \neq 0$, you create four terms. Two of these can usually be simplified by collecting like terms.

Examples

Example 1 Expand $4(3x - 2)$

$4(3x - 2) = 12x - 8$	Multiply everything inside the bracket by the 4 outside the bracket

Example 2 Expand and simplify $3(x + 5) - 4(2x + 3)$

$3(x + 5) - 4(2x + 3)$ $= 3x + 15 - 8x - 12$ $= 3 - 5x$	**1** Expand each set of brackets separately by multiplying $(x + 5)$ by 3 and $(2x + 3)$ by -4 **2** Simplify by collecting like terms: $3x - 8x = -5x$ and $15 - 12 = 3$

Example 3 Expand and simplify $(x + 3)(x + 2)$

$(x + 3)(x + 2)$ $= x(x + 2) + 3(x + 2)$ $= x^2 + 2x + 3x + 6$ $= x^2 + 5x + 6$	**1** Expand the brackets by multiplying $(x + 2)$ by x and $(x + 2)$ by 3 **2** Simplify by collecting like terms: $2x + 3x = 5x$

Example 4 Expand and simplify $(x - 5)(2x + 3)$

$(x - 5)(2x + 3)$ $= x(2x + 3) - 5(2x + 3)$ $= 2x^2 + 3x - 10x - 15$ $= 2x^2 - 7x - 15$	**1** Expand the brackets by multiplying $(2x + 3)$ by x and $(2x + 3)$ by -5 **2** Simplify by collecting like terms: $3x - 10x = -7x$

Practice

1 Expand.

a $3(2x - 1)$ **b** $-2(5pq + 4q^2)$ **c** $-(3xy - 2y^2)$

2 Expand and simplify.

a $7(3x + 5) + 6(2x - 8)$ **b** $8(5p - 2) - 3(4p + 9)$

c $9(3s + 1) - 5(6s - 10)$ **d** $2(4x - 3) - (3x + 5)$

3 Expand.

a $3x(4x + 8)$ **b** $4k(5k^2 - 12)$

c $-2h(6h^2 + 11h - 5)$ **d** $-3s(4s^2 - 7s + 2)$

4 Expand and simplify.

a $3(y^2 - 8) - 4(y^2 - 5)$ **b** $2x(x + 5) + 3x(x - 7)$

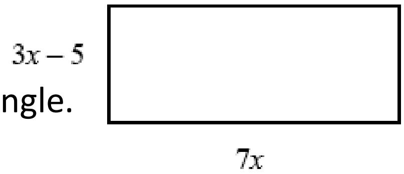
c $4p(2p - 1) - 3p(5p - 2)$ **d** $3b(4b - 3) - b(6b - 9)$

5 Expand $\frac{1}{2}(2y - 8)$

6 Expand and simplify.

a $13 - 2(m + 7)$ **b** $5p(p^2 + 6p) - 9p(2p - 3)$

7 The diagram shows a rectangle.
Write down an expression, in terms of x, for the area of the rectangle.
Show that the area of the rectangle can be written as $21x^2 - 35x$



8 Expand and simplify.

a $(x + 4)(x + 5)$ **b** $(x + 7)(x + 3)$ **c** $(x + 7)(x - 2)$

d $(x + 5)(x - 5)$ **e** $(2x + 3)(x - 1)$ **f** $(3x - 2)(2x + 1)$

g $(5x - 3)(2x - 5)$ **h** $(3x - 2)(7 + 4x)$ **i** $(3x + 4y)(5y + 6x)$

j $(x + 5)^2$ **k** $(2x - 7)^2$ **l** $(4x - 3y)^2$

Extend

9 Expand and simplify $(x + 3)^2 + (x - 4)^2$

10 Expand and simplify.

a $\left(x + \frac{1}{x}\right)\left(x - \frac{2}{x}\right)$ **b** $\left(x + \frac{1}{x}\right)^2$

1.2 Factorising Expressions

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation, find two numbers whose sum is b and whose product is ac.
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

Examples

Example 1 Factorise $15x^2y^3 + 9x^4y$

$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$	The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3 Factorise $x^2 + 3x - 10$

$b = 3$, $ac = -10$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and −2)
So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$	**2** Rewrite the b term ($3x$) using these two factors
$= x(x + 5) - 2(x + 5)$	**3** Factorise the first two terms and the last two terms
$= (x + 5)(x - 2)$	**4** $(x + 5)$ is a factor of both terms

Example 4 Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (−15 and 4)
So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$	**2** Rewrite the b term ($-11x$) using these two factors
$= 3x(2x - 5) + 2(2x - 5)$	**3** Factorise the first two terms and the last two terms
$= (2x - 5)(3x + 2)$	**4** $(2x - 5)$ is a factor of both terms

Example 5 Simplify $\dfrac{x^2-4x-21}{2x^2+9x+9}$

$\dfrac{x^2-4x-21}{2x^2+9x+9}$	**1** Factorise the numerator and the denominator
For the numerator: $b = -4$, $ac = -21$	**2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (−7 and 3)
So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$	**3** Rewrite the b term ($-4x$) using these two factors
$= x(x - 7) + 3(x - 7)$	**4** Factorise the first two terms and the last two terms
$= (x - 7)(x + 3)$	**5** $(x - 7)$ is a factor of both terms
For the denominator: $b = 9$, $ac = 18$	**6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3)
So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$	**7** Rewrite the b term ($9x$) using these two factors
$= 2x(x + 3) + 3(x + 3)$	**8** Factorise the first two terms and the last two terms
$= (x + 3)(2x + 3)$	**9** $(x + 3)$ is a factor of both terms
So $\dfrac{x^2-4x-21}{2x^2+9x+9} = \dfrac{(x-7)(x+3)}{(x+3)(2x+3)}$ $= \dfrac{x-7}{2x+3}$	**10** $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

Practice

1 Factorise.

a $6x^4y^3 - 10x^3y^4$ **b** $21a^3b^5 + 35a^5b^2$ **c** $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

2 Factorise

a $x^2 + 7x + 12$ **b** $x^2 + 5x - 14$ **c** $x^2 - 11x + 30$

d $x^2 - 5x - 24$ **e** $x^2 - 7x - 18$ **f** $x^2 + x - 20$

g $x^2 - 3x - 40$ **h** $x^2 + 3x - 28$

3 Factorise

a $36x^2 - 49y^2$ **b** $4x^2 - 81y^2$ **c** $18a^2 - 200b^2c^2$

4 Factorise

a $2x^2 + x - 3$ **b** $6x^2 + 17x + 5$ **c** $2x^2 + 7x + 3$

d $9x^2 - 15x + 4$ **e** $10x^2 + 21x + 9$ **f** $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a $\frac{2x^2+4x}{x^2-x}$ **b** $\frac{x^2+3x}{x^2+2x-3}$ **c** $\frac{x^2-2x-8}{x^2-4x}$

d $\frac{x^2-5x}{x^2-25}$ **e** $\frac{x^2-x-12}{x^2-4x}$ **f** $\frac{2x^2+14x}{2x^2+4x-70}$

6 Simplify

a $\frac{9x^2-16}{3x^2+17x-28}$ **b** $\frac{2x^2-7x-15}{3x^2-17x+10}$

c $\frac{4-25x^2}{10x^2-11x-6}$ **d** $\frac{6x^2-x-1}{2x^2+7x-4}$

Extend

7 Simplify $\sqrt{x^2 + 10x + 25}$

8 Simplify $\frac{(x+2)^2+3(x+2)^2}{x^2-4}$

1.3 Laws of indices

Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the *n*th root of *a*
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Examples

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1

Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $= 3^2$ $= 9$	**1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ **2** Use $\sqrt[3]{27} = 3$

Example 4 Evaluate 4^{-2}

$4^{-2} = \frac{1}{4^2}$ $= \frac{1}{16}$	**1** Use the rule $a^{-m} = \frac{1}{a^m}$ **2** Use $4^2 = 16$

Example 5 Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$

Example 6 Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	**1** Use the rule $a^m \times a^n = a^{m+n}$ **2** Use the rule $\frac{a^m}{a^n} = a^{m-n}$

Example 7 Write $\frac{1}{3x}$ as a single power of *x*

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged

Example 8 Write $\frac{4}{\sqrt{x}}$ as a single power of *x*

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	**1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ **2** Use the rule $\frac{1}{a^m} = a^{-m}$

Practice

1 Evaluate.

a 14^0 **b** 3^0 **c** 5^0 **d** x^0

2 Evaluate.

a $49^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $125^{\frac{1}{3}}$ **d** $16^{\frac{1}{4}}$

3 Evaluate.

a $25^{\frac{3}{2}}$ **b** $8^{\frac{5}{3}}$ **c** $49^{\frac{3}{2}}$ **d** $16^{\frac{3}{4}}$

4 Evaluate.

a 5^{-2} **b** 4^{-3} **c** 2^{-5} **d** 6^{-2}

5 Simplify.

a $\frac{3x^2 \times x^3}{2x^2}$ **b** $\frac{10x^5}{2x^2 \times x}$ **c** $\frac{3x \times 2x^3}{2x^3}$ **d** $\frac{7x^3y^2}{14x^5y}$

e $\frac{y^2}{y^{\frac{1}{2}} \times y}$ **f** $\frac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$ **g** $\frac{(2x^2)^3}{4x^0}$ **h** $\frac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

6 Evaluate.

a $4^{-\frac{1}{2}}$ **b** $27^{-\frac{2}{3}}$ **c** $9^{-\frac{1}{2}} \times 2^3$

d $16^{\frac{1}{4}} \times 2^{-3}$ **e** $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$ **f** $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$

7 Write the following as a single power of *x*.

a $\frac{1}{x}$ **b** $\frac{1}{x^7}$ **c** $\sqrt[4]{x}$

d $\sqrt[5]{x^2}$ **e** $\frac{1}{\sqrt[3]{x}}$ **f** $\frac{1}{\sqrt[3]{x^2}}$

8 Write the following without negative or fractional powers.

a x^{-3} **b** x^0 **c** $x^{\frac{1}{5}}$

d $x^{\frac{2}{5}}$ **e** $x^{-\frac{1}{2}}$ **f** $x^{-\frac{3}{4}}$

9 Write the following in the form ax^n.

a $5\sqrt{x}$ **b** $\frac{2}{x^3}$ **c** $\frac{1}{3x^4}$

d $\frac{2}{\sqrt{x}}$ **e** $\frac{4}{\sqrt[3]{x}}$ **f** 3

Extend

10 Write as sums of powers of *x*.

a $\frac{x^5+1}{x^2}$ **b** $x^2\left(x + \frac{1}{x}\right)$ **c** $x^{-4}\left(x^2 + \frac{1}{x^3}\right)$

1.4 Surds

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\frac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b - \sqrt{c}$

Examples

Example 1 Simplify $\sqrt{50}$

$\sqrt{50} = \sqrt{25 \times 2}$ $= \sqrt{25} \times \sqrt{2}$ $= 5 \times \sqrt{2}$ $= 5\sqrt{2}$	**1** Choose two numbers that are factors of 50. One of the factors must be a square number **2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ **3** Use $\sqrt{25} = 5$

Example 2 Simplify $\sqrt{147} - 2\sqrt{12}$

$\sqrt{147} - 2\sqrt{12}$ $= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$ $= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$ $= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3}$ $= 7\sqrt{3} - 4\sqrt{3}$ $= 3\sqrt{3}$	**1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number **2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ **3** Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$ **4** Collect like terms

Example 3 Simplify $(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2})$

$(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2})$ $= \sqrt{49} - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - \sqrt{4}$ $= 7 - 2$ $= 5$	**1** Expand the brackets. A common mistake here is to write $(\sqrt{7})^2 = 49$ **2** Collect like terms: $-\sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7}$ $= -\sqrt{7}\sqrt{2} + \sqrt{7}\sqrt{2} = 0$

Example 4 Rationalise $\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{1\times\sqrt{3}}{\sqrt{9}}$ $= \frac{\sqrt{3}}{3}$	**1** Multiply the numerator and denominator by $\sqrt{3}$ **2** Use $\sqrt{9} = 3$

Example 5 Rationalise and simplify $\frac{\sqrt{2}}{\sqrt{12}}$

$\frac{\sqrt{2}}{\sqrt{12}} = \frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$ $= \frac{\sqrt{2}\times\sqrt{4\times3}}{12}$ $= \frac{2\sqrt{2}\sqrt{3}}{12}$ $= \frac{\sqrt{2}\sqrt{3}}{6}$	**1** Multiply the numerator and denominator by $\sqrt{12}$ **2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number **3** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ **4** Use $\sqrt{4} = 2$ **5** Simplify the fraction: $\frac{2}{12}$ simplifies to $\frac{1}{6}$

Example 6 Rationalise and simplify $\frac{3}{2+\sqrt{5}}$

$\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$ $= \frac{3(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$ $= \frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$ $= \frac{6-3\sqrt{5}}{-1}$ $= 3\sqrt{5} - 6$	**1** Multiply the numerator and denominator by $2-\sqrt{5}$ **2** Expand the brackets **3** Simplify the fraction **4** Divide the numerator by −1 Remember to change the sign of all terms when dividing by −1

Practice

1 Simplify.

a $\sqrt{45}$ **b** $\sqrt{125}$ **c** $\sqrt{48}$ **d** $\sqrt{175}$

e $\sqrt{300}$ **f** $\sqrt{28}$ **g** $\sqrt{72}$ **h** $\sqrt{162}$

2 Simplify.

a $\sqrt{72} + \sqrt{162}$ **b** $\sqrt{45} - 2\sqrt{5}$ **c** $\sqrt{50} - \sqrt{8}$

d $\sqrt{75} - \sqrt{48}$ **e** $2\sqrt{28} + \sqrt{28}$ **f** $2\sqrt{12} - \sqrt{12} + \sqrt{27}$

3 Expand and simplify.

a $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$ **b** $(3 + \sqrt{3})(5 - \sqrt{12})$

c $(4 - \sqrt{5})(\sqrt{45} + 2)$ **d** $(5 + \sqrt{2})(6 - \sqrt{8})$

4 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{5}}$ **b** $\frac{1}{\sqrt{11}}$ **c** $\frac{2}{\sqrt{7}}$

d $\frac{2}{\sqrt{8}}$ **e** $\frac{2}{\sqrt{2}}$ **f** $\frac{5}{\sqrt{5}}$

g $\frac{\sqrt{8}}{\sqrt{24}}$ **h** $\frac{\sqrt{5}}{\sqrt{45}}$

5 Rationalise and simplify.

a $\frac{1}{3 - \sqrt{5}}$ **b** $\frac{2}{4 + \sqrt{3}}$ **c** $\frac{6}{5 - \sqrt{2}}$

Extend

6 Expand and simplify $\left(\sqrt{x} + \sqrt{y}\right)\left(\sqrt{x} - \sqrt{y}\right)$

7 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{9} - \sqrt{8}}$ **b** $\frac{1}{\sqrt{x} - \sqrt{y}}$

2 Solving Quadratic Equations

2.1 Factorising

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation, find two numbers whose sum is b and whose products is ac.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Examples

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$	**1** Rearrange the equation so that all the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$.
$5x(x - 3) = 0$	**2** Factorise the quadratic equation. $5x$ is a common factor.
So $5x = 0$ or $(x - 3) = 0$	**3** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = 0$ or $x = 3$	**4** Solve these two equations.

Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7$, $ac = 12$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3)
$x^2 + 4x + 3x + 12 = 0$	**2** Rewrite the b term ($7x$) using these two factors.
$x(x + 4) + 3(x + 4) = 0$	**3** Factorise the first two terms and the last two terms.
$(x + 4)(x + 3) = 0$	**4** $(x + 4)$ is a factor of both terms.
So $(x + 4) = 0$ or $(x + 3) = 0$	**5** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = -4$ or $x = -3$	**6** Solve these two equations.

Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$ So $(3x + 4) = 0$ or $(3x - 4) = 0$ $x = -\frac{4}{3}$ or $x = \frac{4}{3}$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$. **2** When two values multiply to make zero, at least one of the values must be zero. **3** Solve these two equations.

Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5$, $ac = -24$ So $2x^2 - 8x + 3x - 12 = 0$ $2x(x - 4) + 3(x - 4) = 0$ $(x - 4)(2x + 3) = 0$ So $(x - 4) = 0$ or $(2x + 3) = 0$ $x = 4$ or $x = -\frac{3}{2}$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (-8 and 3) **2** Rewrite the b term ($-5x$) using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x - 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Practice

1 Solve

a $6x^2 + 4x = 0$ **b** $28x^2 - 21x = 0$ **c** $x^2 + 7x + 10 = 0$

d $x^2 - 5x + 6 = 0$ **e** $x^2 - 3x - 4 = 0$ **f** $x^2 + 3x - 10 = 0$

g $x^2 - 10x + 24 = 0$ **h** $x^2 - 36 = 0$ **i** $x^2 + 3x - 28 = 0$

j $x^2 - 6x + 9 = 0$ **k** $2x^2 - 7x - 4 = 0$ **l** $3x^2 - 13x - 10 = 0$

2 Solve

a $x^2 - 3x = 10$ **b** $x^2 - 3 = 2x$ **c** $x^2 + 5x = 24$

d $x^2 - 42 = x$ **e** $x(x + 2) = 2x + 25$ **f** $x^2 - 30 = 3x - 2$

g $x(3x + 1) = x^2 + 15$ **h** $3x(x - 1) = 2(x + 1)$

2.1 Using the formula

Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- If $b^2 - 4ac$ is negative, then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for a, b and c.

Examples

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1, b = 6, c = 4$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$	**2** Substitute $a = 1$, $b = 6$, $c = 4$ into the formula.
$x = \dfrac{-6 \pm \sqrt{20}}{2}$	**3** Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.
$x = \dfrac{-6 \pm 2\sqrt{5}}{2}$	**4** Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
$x = -3 \pm \sqrt{5}$	**5** Simplify by dividing numerator and denominator by 2.
So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	**6** Write down both the solutions.

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3, b = -7, c = -2$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$	**2** Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.
$x = \dfrac{7 \pm \sqrt{73}}{6}$	**3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.
So $x = \frac{7 - \sqrt{73}}{6}$ or $x = \frac{7 + \sqrt{73}}{6}$	**4** Write down both the solutions.

Practice

1 Solve, giving your solutions in surd form.

a $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

2 Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\frac{a \pm \sqrt{b}}{c}$, where a, b and c are integers.

3 Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

Extend

4 Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

a $4x(x - 1) = 3x - 2$

b $10 = (x + 1)^2$

c $x(3x - 1) = 10$

3 Simultaneous Equations

3.1 Linear - Elimination

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$$\begin{array}{rl} & 3x + y = 5 \\ - & \underline{\;\;x + y = 1} \\ & 2x \quad\;\; = 4 \end{array}$$ So $x = 2$	**1** Subtract the second equation from the first equation to eliminate the y term.
Using $x + y = 1$ $2 + y = 1$ So $y = −1$	**2** To find the value of y, substitute $x = 2$ into one of the original equations.
Check: equation 1: $3 × 2 + (−1) = 5$ YES equation 2: $2 + (−1) = 1$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $x + 2y = 13$ and $5x − 2y = 5$ simultaneously.

$$\begin{array}{rl} & x + 2y = 13 \\ + & \underline{5x - 2y = \;\;5} \\ & 6x \qquad = 18 \end{array}$$ So $x = 3$	**1** Add the two equations together to eliminate the y term.
Using $x + 2y = 13$ $3 + 2y = 13$ So $y = 5$	**2** To find the value of y, substitute $x = 3$ into one of the original equations.
Check: equation 1: $3 + 2 × 5 = 13$ YES equation 2: $5 × 3 − 2 × 5 = 5$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 3 Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$(2x + 3y = 2) \times 4 \rightarrow \quad 8x + 12y = 8$ $(5x + 4y = 12) \times 3 \rightarrow \quad \underline{15x + 12y = 36}$ $7x \quad = 28$ So $x = 4$	**1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term.
Using $2x + 3y = 2$ $2 \times 4 + 3y = 2$ So $y = -2$	**2** To find the value of y, substitute $x = 4$ into one of the original equations.
Check: equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $4x + y = 8$
$x + y = 5$

2 $3x + y = 7$
$3x + 2y = 5$

3 $4x + y = 3$
$3x - y = 11$

4 $3x + 4y = 7$
$x - 4y = 5$

5 $2x + y = 11$
$x - 3y = 9$

6 $2x + 3y = 11$
$3x + 2y = 4$

3.2 Linear – Substitution

Key points

- The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

Example 1 Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

$5x + 3(2x + 1) = 14$ $5x + 6x + 3 = 14$ $11x + 3 = 14$ $11x = 11$ So $x = 1$	**1** Substitute $2x + 1$ for y into the second equation. **2** Expand the brackets and simplify. **3** Work out the value of x.
Using $y = 2x + 1$ $y = 2 \times 1 + 1$ So $y = 3$	**4** To find the value of y, substitute $x = 1$ into one of the original equations.
Check: equation 1: $3 = 2 \times 1 + 1$ YES equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	**5** Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

$y = 2x - 16$ $4x + 3(2x - 16) = -3$ $4x + 6x - 48 = -3$ $10x - 48 = -3$ $10x = 45$ So $x = 4\frac{1}{2}$	**1** Rearrange the first equation. **2** Substitute $2x - 16$ for y into the second equation. **3** Expand the brackets and simplify. **4** Work out the value of x.
Using $y = 2x - 16$ $y = 2 \times 4\frac{1}{2} - 16$ So $y = -7$	**5** To find the value of y, substitute $x = 4\frac{1}{2}$ into one of the original equations.
Check: equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES	**6** Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $y = x - 4$
$2x + 5y = 43$

2 $y = 2x - 3$
$5x - 3y = 11$

3 $2y = 4x + 5$
$9x + 5y = 22$

4 $2x = y - 2$
$8x - 5y = -11$

5 $3x + 4y = 8$
$2x - y = -13$

6 $3y = 4x - 7$
$2y = 3x - 4$

7 $3x = y - 1$
$2y - 2x = 3$

8 $3x + 2y + 1 = 0$
$4y = 8 - x$

Extend

9 Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x + y) = \frac{3(y-x)}{4}$.

3.3 Linear and Quadratic

Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Examples

Example 1 Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$

$x^2 + (x + 1)^2 = 13$ $x^2 + x^2 + x + x + 1 = 13$ $2x^2 + 2x + 1 = 13$ $2x^2 + 2x - 12 = 0$ $(2x - 4)(x + 3) = 0$ So $x = 2$ or $x = -3$ Using $y = x + 1$ When $x = 2$, $y = 2 + 1 = 3$ When $x = -3$, $y = -3 + 1 = -2$ So the solutions are $x = 2,\ y = 3$ and $x = -3, y = -2$ Check: equation 1: $3 = 2 + 1$ YES and $-2 = -3 + 1$ YES equation 2: $2^2 + 3^2 = 13$ YES and $(-3)^2 + (-2)^2 = 13$ YES	**1** Substitute $x + 1$ for y into the second equation. **2** Expand the brackets and simplify. **3** Factorise the quadratic equation. **4** Work out the values of x. **5** To find the value of y, substitute both values of x into one of the original equations. **6** Substitute both pairs of values of x and y into both equations to check your answers.

Example 2 Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

$x = \frac{5-3y}{2}$	**1** Rearrange the first equation.
$2y^2 + \left(\frac{5-3y}{2}\right)y = 12$	**2** Substitute $\frac{5-3y}{2}$ for x into the second equation. Notice how it is easier to substitute for x than for y.
$2y^2 + \frac{5y-3y^2}{2} = 12$ $4y^2 + 5y - 3y^2 = 24$ $y^2 + 5y - 24 = 0$	**3** Expand the brackets and simplify.
$(y + 8)(y - 3) = 0$	**4** Factorise the quadratic equation.
So $y = -8$ or $y = 3$	**5** Work out the values of y.
Using $2x + 3y = 5$ When $y = -8$, $2x + 3 \times (-8) = 5$, $x = 14.5$ When $y = 3$, $2x + 3 \times 3 = 5$, $x = -2$	**6** To find the value of x, substitute both values of y into one of the original equations.
So the solutions are $x = 14.5$, $y = -8$ and $x = -2$, $y = 3$	
Check: equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES and $2 \times (-2) + 3 \times 3 = 5$ YES equation 2: $2\times(-8)^2 + 14.5\times(-8) = 12$ YES and $2 \times (3)^2 + (-2) \times 3 = 12$ YES	**7** Substitute both pairs of values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $y = 2x + 1$
$x^2 + y^2 = 10$

2 $y = 6 - x$
$x^2 + y^2 = 20$

3 $y = x - 3$
$x^2 + y^2 = 5$

4 $y = 9 - 2x$
$x^2 + y^2 = 17$

5 $y = 3x - 5$
$y = x^2 - 2x + 1$

6 $y = x - 5$
$y = x^2 - 5x - 12$

7 $y = x + 5$
$x^2 + y^2 = 25$

8 $y = 2x - 1$
$x^2 + xy = 24$

9 $y = 2x$
$y^2 - xy = 8$

10 $2x + y = 11$
$xy = 15$

Extend

11 $x - y = 1$
$x^2 + y^2 = 3$

12 $y - x = 2$
$x^2 + xy = 3$

4 Linear inequalities

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

Examples

Example 1 Solve $-8 \le 4x < 16$

$-8 \le 4x < 16$ $-2 \le x < 4$	Divide all three terms by 4.

Example 2 Solve $4 \le 5x < 10$

$4 \le 5x < 10$ $\frac{4}{5} \le x < 2$	Divide all three terms by 5.

Example 3 Solve $2x - 5 < 7$

$2x - 5 < 7$ $2x < 12$ $x < 6$	**1** Add 5 to both sides. **2** Divide both sides by 2.

Example 4 Solve $2 - 5x \ge -8$

$2 - 5x \ge -8$ $-5x \ge -10$ $x \le 2$	**1** Subtract 2 from both sides. **2** Divide both sides by −5. Remember to reverse the inequality when dividing by a negative number.

Example 5 Solve $4(x - 2) > 3(9 - x)$

$4(x - 2) > 3(9 - x)$ $4x - 8 > 27 - 3x$ $7x - 8 > 27$ $7x > 35$ $x > 5$	**1** Expand the brackets. **2** Add $3x$ to both sides. **3** Add 8 to both sides. **4** Divide both sides by 7.

Practice

1 Solve these inequalities.

a $4x > 16$ **b** $5x - 7 \leq 3$ **c** $1 \geq 3x + 4$

d $5 - 2x < 12$ **e** $\frac{x}{2} \geq 5$ **f** $8 < 3 - \frac{x}{3}$

2 Solve these inequalities.

a $\frac{x}{5} < -4$ **b** $10 \geq 2x + 3$ **c** $7 - 3x > -5$

3 Solve

a $2 - 4x \geq 18$ **b** $3 \leq 7x + 10 < 45$ **c** $6 - 2x \geq 4$

d $4x + 17 < 2 - x$ **e** $4 - 5x < -3x$ **f** $-4x \geq 24$

4 Solve these inequalities.

a $3t + 1 < t + 6$ **b** $2(3n - 1) \geq n + 5$

5 Solve.

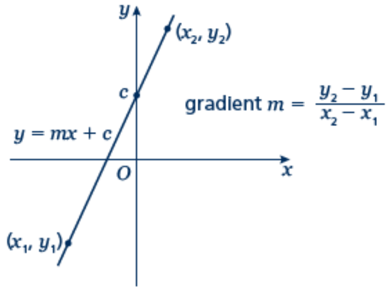
a $3(2 - x) > 2(4 - x) + 4$ **b** $5(4 - x) > 3(5 - x) + 2$

Extend

6 Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

5 Straight line Graphs

5.1 Equations



Key points

- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y-intercept
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a, b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

Examples

Example 1 A straight line has gradient $-\frac{1}{2}$ and y-intercept 3.

Write the equation of the line in the form $ax + by + c = 0$.

$m = -\frac{1}{2}$ and $c = 3$ So $y = -\frac{1}{2}x + 3$ $\frac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	**1** A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation. **2** Rearrange the equation so all the terms are on one side and 0 is on the other side. **3** Multiply both sides by 2 to eliminate the denominator.

Example 2 Find the gradient and the y-intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \frac{2}{3}x - \frac{4}{3}$ Gradient $= m = \frac{2}{3}$ y-intercept $= c = -\frac{4}{3}$	**1** Make y the subject of the equation. **2** Divide all the terms by three to get the equation in the form $y = \ldots$ **3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$ $13 = 3 \times 5 + c$ $13 = 15 + c$ $c = -2$ $y = 3x - 2$	**1** Substitute the gradient given in the question into the equation of a straight line $y = mx + c$. **2** Substitute the coordinates $x = 5$ and $y = 13$ into the equation. **3** Simplify and solve the equation. **4** Substitute $c = -2$ into the equation $y = 3x + c$

Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2$, $x_2 = 8$, $y_1 = 4$ and $y_2 = 7$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-4}{8-2} = \frac{3}{6} = \frac{1}{2}$	**1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$y = \frac{1}{2}x + c$	**2** Substitute the gradient into the equation of a straight line *y* = *mx* + *c*.
$4 = \frac{1}{2} \times 2 + c$	**3** Substitute the coordinates of either point into the equation.
c = 3	**4** Simplify and solve the equation.
$y = \frac{1}{2}x + 3$	**5** Substitute *c* = 3 into the equation $y = \frac{1}{2}x + c$

Practice

1 Find the gradient and the *y*-intercept of the following equations.

a $y = 3x + 5$ **b** $y = -\frac{1}{2}x - 7$ **c** $2y = 4x - 3$

d $x + y = 5$ **e** $2x - 3y - 7 = 0$ **f** $5x + y - 4 = 0$

2 Find, in the form $ax + by + c = 0$ where *a*, *b* and *c* are integers, an equation for each of the lines with the following gradients and *y*-intercepts.

a gradient $-\frac{1}{2}$, *y*-intercept –7 **b** gradient 2, *y*-intercept 0

c gradient $\frac{2}{3}$, *y*-intercept 4 **d** gradient –1.2, *y*-intercept –2

3 Write an equation for the line which passes though the point (2, 5) and has gradient 4.

4 Write an equation for the line which passes through the point (6, 3) and has gradient $-\frac{2}{3}$

5 Write an equation for the line passing through each of the following pairs of points.

a (4, 5), (10, 17) **b** (0, 6), (–4, 8)

c (–1, –7), (5, 23) **d** (3, 10), (4, 7)

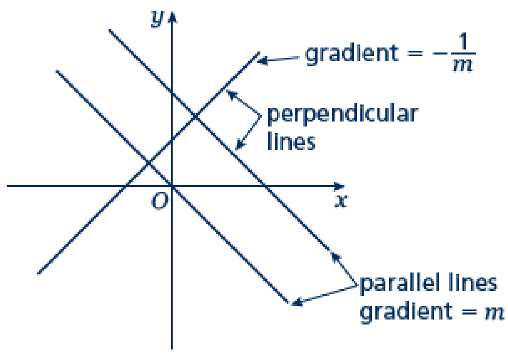
Extend

6 The equation of a line is $2y + 3x - 6 = 0$.
Write as much information as possible about this line.

5.2 Parallel and Perpendicular

Key points

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation $y = mx + c$ has gradient $-\frac{1}{m}$.



Examples

Example 1 Find the equation of the line parallel to $y = 2x + 4$ which passes through the point (4, 9).

$y = 2x + 4$ $m = 2$ $y = 2x + c$ $9 = 2 \times 4 + c$ $9 = 8 + c$ $c = 1$ $y = 2x + 1$	**1** As the lines are parallel they have the same gradient. **2** Substitute $m = 2$ into the equation of a straight line $y = mx + c$. **3** Substitute the coordinates into the equation $y = 2x + c$ **4** Simplify and solve the equation. **5** Substitute $c = 1$ into the equation $y = 2x + c$

Example 2 Find the equation of the line perpendicular to $y = 2x - 3$ which passes through the point (−2, 5).

$y = 2x - 3$ $m = 2$ $-\frac{1}{m} = -\frac{1}{2}$ $y = -\frac{1}{2}x + c$ $5 = -\frac{1}{2} \times (-2) + c$ $5 = 1 + c$ $c = 4$ $y = -\frac{1}{2}x + 4$	**1** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$. **2** Substitute $m = -\frac{1}{2}$ into $y = mx + c$. **3** Substitute the coordinates (–2, 5) into the equation $y = -\frac{1}{2}x + c$ **4** Simplify and solve the equation. **5** Substitute $c = 4$ into $y = -\frac{1}{2}x + c$.

Example 3 A line passes through the points (0, 5) and (9, −1).
Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$x_1 = 0, x_2 = 9, y_1 = 5$ and $y_2 = -1$	**1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-5}{9-0}$ $= \dfrac{-6}{9} = -\dfrac{2}{3}$	
$-\dfrac{1}{m} = \dfrac{3}{2}$	**2** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$.
$y = \frac{3}{2}x + c$	**3** Substitute the gradient into the equation $y = mx + c$.
Midpoint = $\left(\frac{0+9}{2}, \frac{5+(-1)}{2}\right) = \left(\frac{9}{2}, 2\right)$	**4** Work out the coordinates of the midpoint of the line.
$2 = \dfrac{3}{2} \times \dfrac{9}{2} + c$	**5** Substitute the coordinates of the midpoint into the equation.
$c = -\frac{19}{4}$	**6** Simplify and solve the equation.
$y = \dfrac{3}{2}x - \dfrac{19}{4}$	**7** Substitute $c = -\dfrac{19}{4}$ into the equation $y = \dfrac{3}{2}x + c$.

Practice

1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

a $y = 3x + 1$ (3, 2) **b** $y = 3 - 2x$ (1, 3)

c $2x + 4y + 3 = 0$ (6, −3) **d** $2y - 3x + 2 = 0$ (8, 20)

2 Find the equation of the line perpendicular to $y = \frac{1}{2}x - 3$ which passes through the point (−5, 3).

3 Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

a $y = 2x - 6$ (4, 0) **b** $y = -\frac{1}{3}x + \frac{1}{2}$ (2, 13)

c $x - 4y - 4 = 0$ (5, 15) **d** $5y + 2x - 5 = 0$ (6, 7)

4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

a (4, 3), (–2, –9) **b** (0, 3), (–10, 8)

Extend

5 Work out whether these pairs of lines are parallel, perpendicular or neither.

a $y = 2x + 3$
$y = 2x - 7$

b $y = 3x$
$2x + y - 3 = 0$

c $y = 4x - 3$
$4y + x = 2$

d $3x - y + 5 = 0$
$x + 3y = 1$

e $2x + 5y - 1 = 0$
$y = 2x + 7$

f $2x - y = 6$
$6x - 3y + 3 = 0$

6 The straight line $\mathbf{L_1}$ passes through the points A and B with coordinates (–4, 4) and (2, 1), respectively.

a Find the equation of $\mathbf{L_1}$ in the form $ax + by + c = 0$

The line $\mathbf{L_2}$ is parallel to the line $\mathbf{L_1}$ and passes through the point C with coordinates (–8, 3).

b Find the equation of $\mathbf{L_2}$ in the form $ax + by + c = 0$

The line $\mathbf{L_3}$ is perpendicular to the line $\mathbf{L_1}$ and passes through the origin.

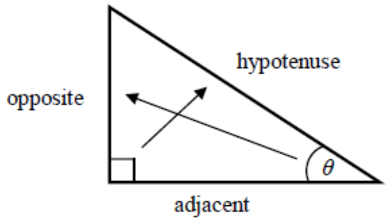
c Find an equation of $\mathbf{L_3}$

6 Trigonometry

6.1 Right-angled

Key points

- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle θ is called the opposite
  - the side next to the angle θ is called the adjacent.

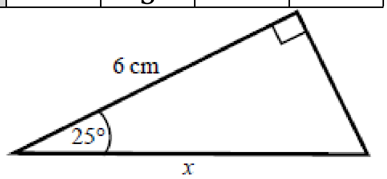


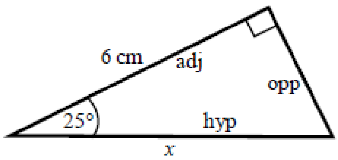
- In a right-angled triangle:
  - the ratio of the opposite side to the hypotenuse is the sine of angle θ, $\sin\theta = \frac{\text{opp}}{\text{hyp}}$
  - the ratio of the adjacent side to the hypotenuse is the cosine of angle θ, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$
  - the ratio of the opposite side to the adjacent side is the tangent of angle θ, $\tan\theta = \frac{\text{opp}}{\text{adj}}$
- The sine, cosine and tangent of some angles may be written exactly.

	0	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

Examples

Example 1 Calculate the length of side *x*.
Give your answer correct to 3 significant figures.





1 Always start by labelling the sides.

$cos\ \theta = \frac{\text{adj}}{\text{hyp}}$

2 You are given the adjacent and the hypotenuse so use the cosine ratio.

$cos\ 25° = \frac{6}{x}$

3 Substitute the sides and angle into the cosine ratio.

$x = \frac{6}{cos\ 25°}$

4 Rearrange to make *x* the subject.

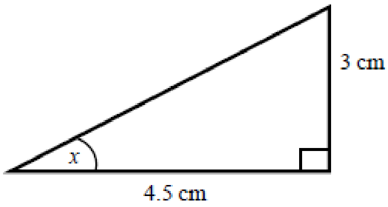
x = 6.620 267 5...

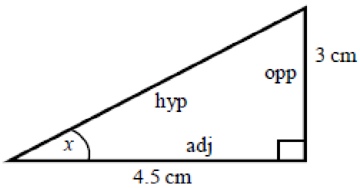
5 Use your calculator to work out 6 ÷ cos 25°.

x = 6.62 cm

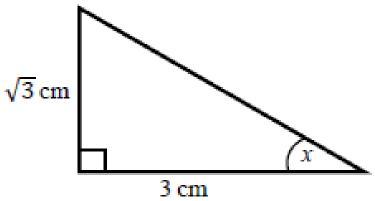
6 Round your answer to 3 significant figures and write the units in your answer.

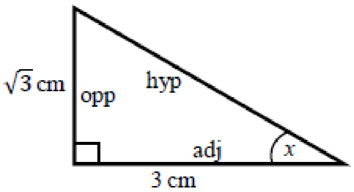
Example 2 Calculate the size of angle x.
Give your answer correct to 3 significant figures.



 $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$ $\tan x = \dfrac{3}{4.5}$ $x = \tan^{-1}\left(\dfrac{3}{4.5}\right)$ $x = 33.690\,067\,5...$ $x = 33.7°$	**1** Always start by labelling the sides. **2** You are given the opposite and the adjacent so use the tangent ratio. **3** Substitute the sides and angle into the tangent ratio. **4** Use $\tan^{-1}$ to find the angle. **5** Use your calculator to work out $\tan^{-1}(3 \div 4.5)$. **6** Round your answer to 3 significant figures and write the units in your answer.

Example 3 Calculate the exact size of angle x.

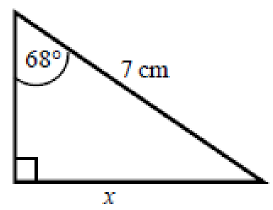


 $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$ $\tan x = \dfrac{\sqrt{3}}{3}$ $x = 30°$	**1** Always start by labelling the sides. **2** You are given the opposite and the adjacent so use the tangent ratio. **3** Substitute the sides and angle into the tangent ratio. **4** Use the table from the key points to find the angle.

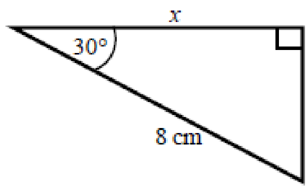
Practice

1 Calculate the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

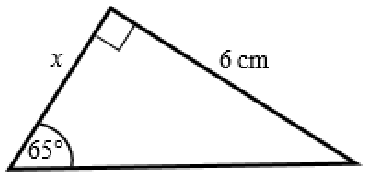
a



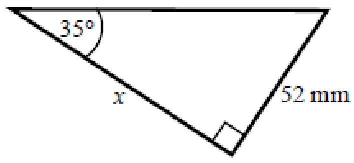
b



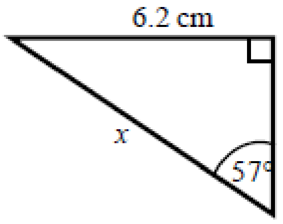
c



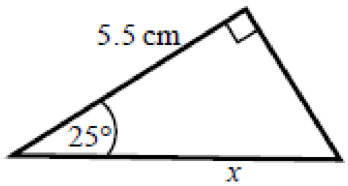
d



e

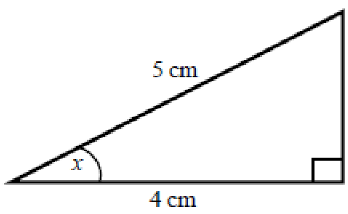


f

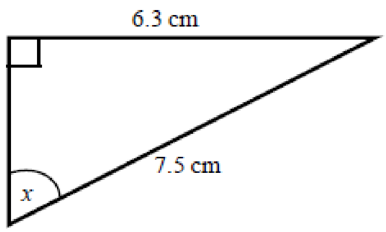


2 Calculate the size of angle *x* in each triangle.
Give your answers correct to 1 decimal place.

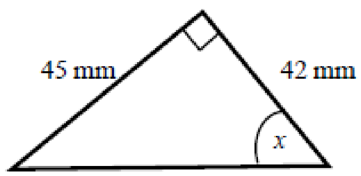
a



b



c



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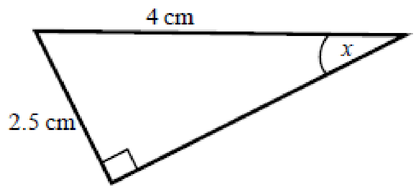
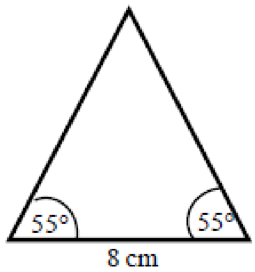
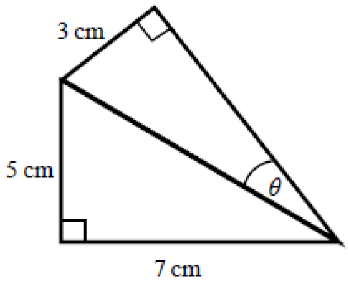


3 Work out the height of the isosceles triangle.
Give your answer correct to 3 significant figures.

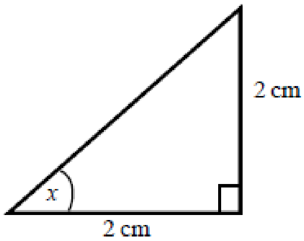


4 Calculate the size of angle θ.
Give your answer correct to 1 decimal place.

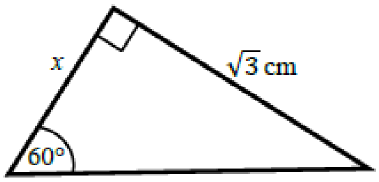


5 Find the exact value of *x* in each triangle.

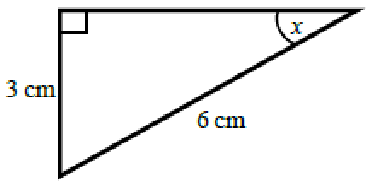
a



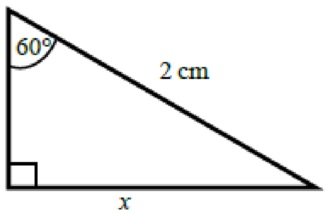
b



c



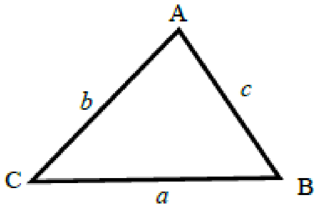
d



6.2 Cosine Rule

Key points

- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

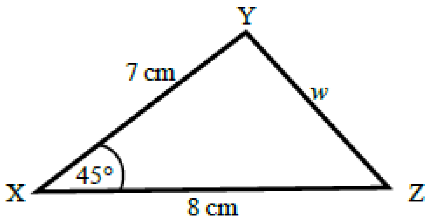


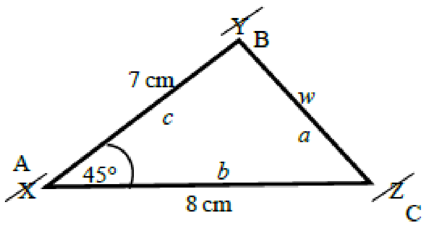
- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc \cos A$.

- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \frac{b^2+c^2-a^2}{2bc}$.

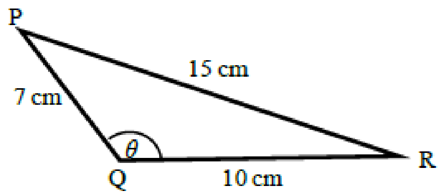
Examples

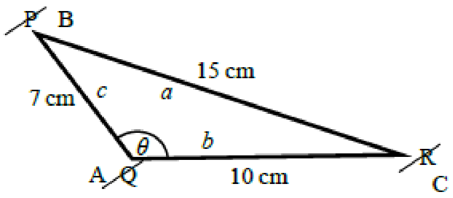
Example 4 Work out the length of side *w*.
Give your answer correct to 3 significant figures.



 $a^2 = b^2 + c^2 - 2bc \cos A$ $w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45°$ w^2 = 33.804 040 51... $w = \sqrt{33.80404051}$ w = 5.81 cm	**1** Always start by labelling the angles and sides. **2** Write the cosine rule to find the side. **3** Substitute the values *a*, *b* and *A* into the formula. **4** Use a calculator to find w^2 and then *w*. **5** Round your final answer to 3 significant figures and write the units in your answer.

Example 5 Work out the size of angle θ.
Give your answer correct to 1 decimal place.

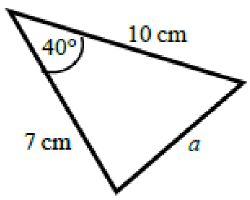


	**1** Always start by labelling the angles and sides.
$cos\ A = \dfrac{b^2 + c^2 - a^2}{2bc}$	**2** Write the cosine rule to find the angle.
$cos\ \theta = \dfrac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$	**3** Substitute the values *a*, *b* and *c* into the formula.
$cos\ \theta = \dfrac{-76}{140}$	**4** Use $\cos^{-1}$ to find the angle.
θ = 122.878 349...	**5** Use your calculator to work out $\cos^{-1}(-76 \div 140)$.
θ = 122.9°	**6** Round your answer to 1 decimal place and write the units in your answer.

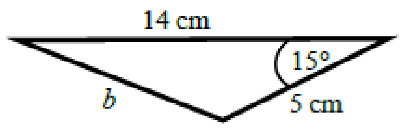
Practice

1 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

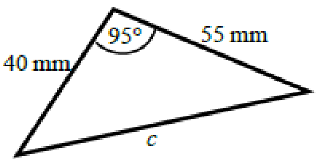
a



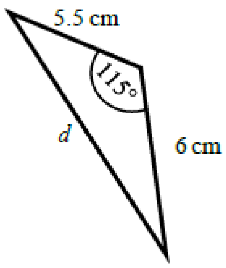
b



c

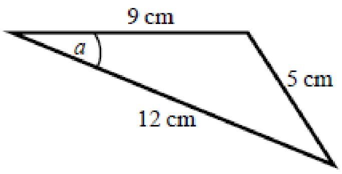


d

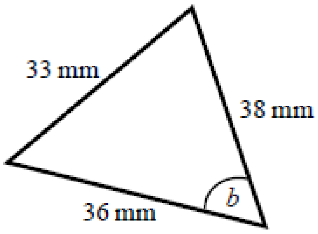


2 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal plac

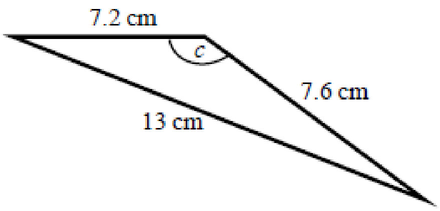
a



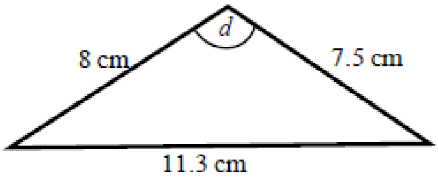
b



c

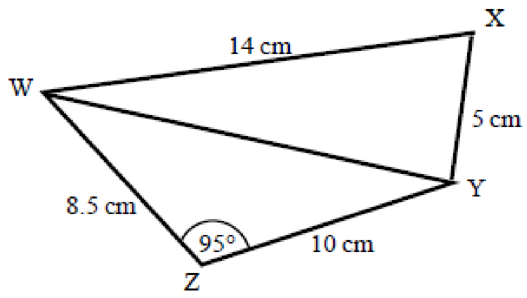


d



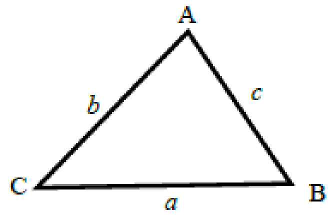
3 **a** Work out the length of WY.
Give your answer correct to
3 significant figures.

b Work out the size of angle WXY.
Give your answer correct to
1 decimal place.



6.3 Sine Rule

Key points

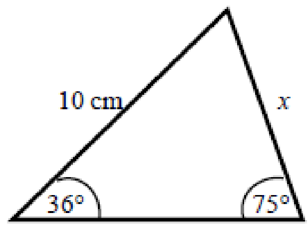


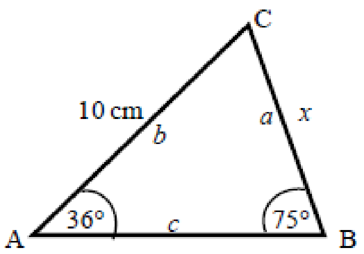
- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

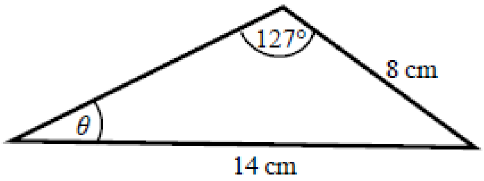
Examples

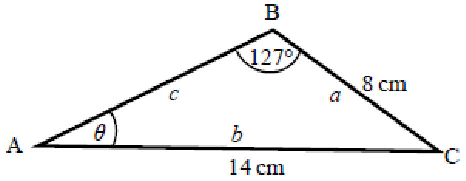
Example 6 Work out the length of side *x*.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the angles and sides.
$\frac{a}{\sin A} = \frac{b}{\sin B}$	**2** Write the sine rule to find the side.
$\frac{x}{\sin 36°} = \frac{10}{\sin 75°}$	**3** Substitute the values *a*, *b*, *A* and *B* into the formula.
$x = \frac{10 \times \sin 36°}{\sin 75°}$	**4** Rearrange to make *x* the subject.
x = 6.09 cm	**5** Round your answer to 3 significant figures and write the units in your answer.

Example 7 Work out the size of angle θ.
Give your answer correct to 1 decimal place.

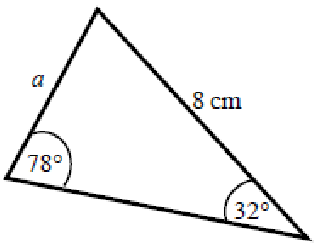


	**1** Always start by labelling the angles and sides.
$\frac{\sin A}{a} = \frac{\sin B}{b}$	**2** Write the sine rule to find the angle.
$\frac{\sin\theta}{8} = \frac{\sin 127°}{14}$	**3** Substitute the values *a*, *b*, *A* and *B* into the formula.
$\sin\theta = \frac{8 \times \sin 127°}{14}$	**4** Rearrange to make sin θ the subject.
θ = 27.2°	**5** Use $\sin^{-1}$ to find the angle. Round your answer to 1 decimal place and write the units in your answer.

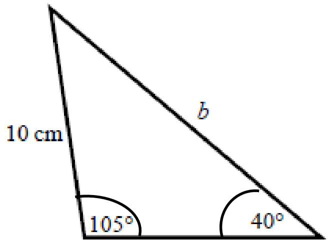
Practice

1 Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

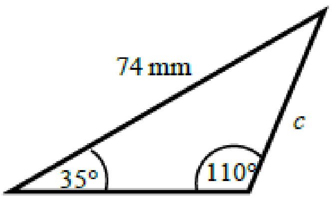
a



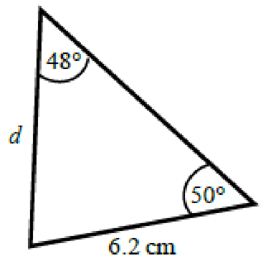
b



c

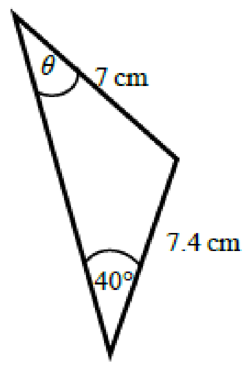


d

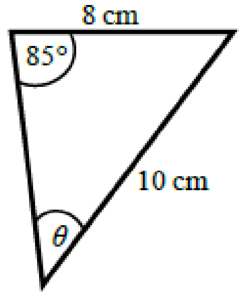


2 Calculate the angles labelled ϑ in each triangle.
Give your answer correct to 1 decimal place.

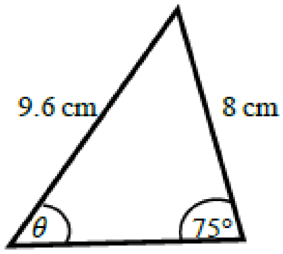
a



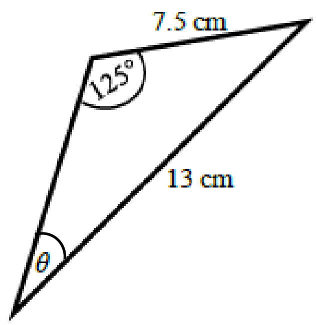
b



c

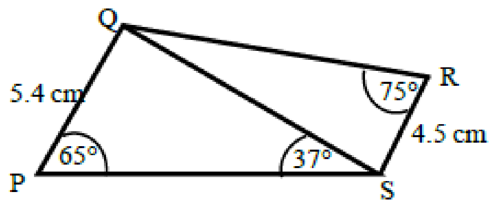


d



3 **a** Work out the length of QS.
Give your answer correct to 3 significant figures.

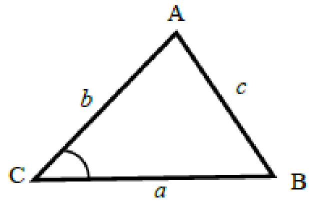
b Work out the size of angle RQS.
Give your answer correct to 1 decimal place.



6.4 Area of a triangle

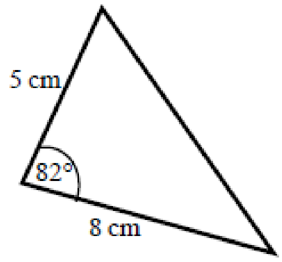
Key points

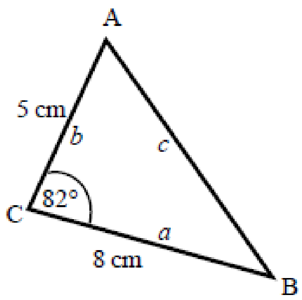
- a is the side opposite angle A. b is the side opposite angle B. c is the side opposite angle C.
- The area of the triangle is $\frac{1}{2}ab\sin C$.



Examples

Example 8 Find the area of the triangle.

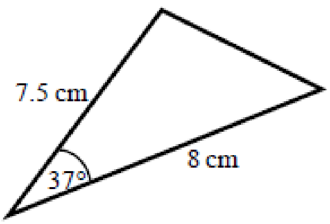


	**1** Always start by labelling the sides and angles of the triangle.
Area = $\frac{1}{2}ab\sin C$	**2** State the formula for the area of a triangle.
Area = $\frac{1}{2} \times 8 \times 5 \times \sin 82°$	**3** Substitute the values of a, b and C into the formula for the area of a triangle.
Area = 19.805 361...	**4** Use a calculator to find the area.
Area = 19.8 cm^2	**5** Round your answer to 3 significant figures and write the units in your answer.

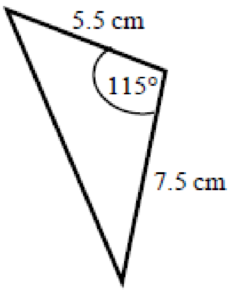
Practice

1 Work out the area of each triangle.
Give your answers correct to 3 significant figures.

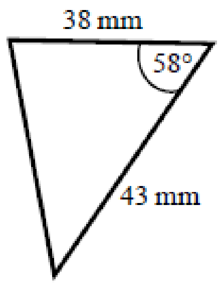
a



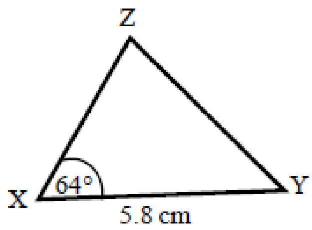
b



c



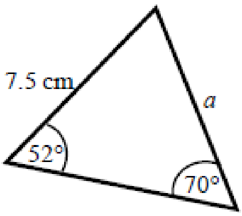
2 The area of triangle XYZ is 13.3 cm^2.
Work out the length of XZ.



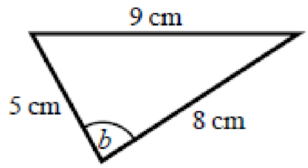
Extend

3 Find the size of each lettered angle or side.
Give your answers correct to 3 significant figures.

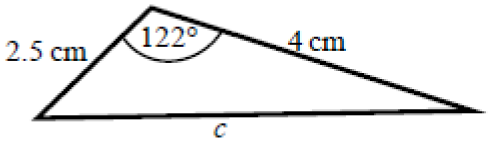
a



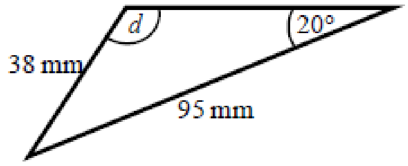
b



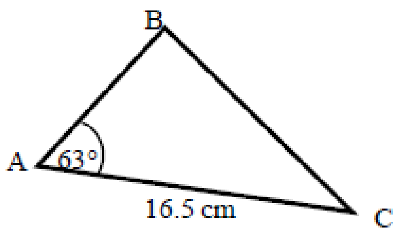
c



d



4 The area of triangle ABC is 86.7 cm^2.
Work out the length of BC.
Give your answer correct to 3 significan



Answers

1 Algebraic Expressions

1.1 Expanding brackets

1 **a** $6x - 3$ **b** $-10pq - 8q^2$ **c** $-3xy + 2y^2$

2 **a** $21x + 35 + 12x - 48 = 33x - 13$ **b** $40p - 16 - 12p - 27 = 28p - 43$
c $27s + 9 - 30s + 50 = -3s + 59 = 59 - 3s$ **d** $8x - 6 - 3x - 5 = 5x - 11$

3 **a** $12x^2 + 24x$ **b** $20k^3 - 48k$
c $10h - 12h^3 - 22h^2$ **d** $21s^2 - 21s^3 - 6s$

4 **a** $-y^2 - 4$ **b** $5x^2 - 11x$
c $2p - 7p^2$ **d** $6b^2$

5 $y - 4$

6 **a** $-1 - 2m$ **b** $5p^3 + 12p^2 + 27p$

7 $7x(3x - 5) = 21x^2 - 35x$

8 **a** $x^2 + 9x + 20$ **b** $x^2 + 10x + 21$
c $x^2 + 5x - 14$ **d** $x^2 - 25$
e $2x^2 + x - 3$ **f** $6x^2 - x - 2$
g $10x^2 - 31x + 15$ **h** $12x^2 + 13x - 14$
i $18x^2 + 39xy + 20y^2$ **j** $x^2 + 10x + 25$
k $4x^2 - 28x + 49$ **l** $16x^2 - 24xy + 9y^2$

9 $2x^2 - 2x + 25$

10 **a** $x^2 - 1 - \frac{2}{x^2}$ **b** $x^2 + 2 + \frac{1}{x^2}$

1.2 Factorising Expressions

1 **a** $2x^3y^3(3x - 5y)$ **b** $7a^3b^2(3b^3 + 5a^2)$ **c** $5x^2y^2(5 - 2x + 3y)$

2 **a** $(x + 3)(x + 4)$ **b** $(x + 7)(x - 2)$ **c** $(x - 5)(x - 6)$

d $(x - 8)(x + 3)$ **e** $(x - 9)(x + 2)$ **f** $(x + 5)(x - 4)$
g $(x - 8)(x + 5)$ **h** $(x + 7)(x - 4)$

3 **a** $(6x - 7y)(6x + 7y)$ **b** $(2x - 9y)(2x + 9y)$ **c** $2(3a - 10bc)(3a + 10bc)$

4 **a** $(x - 1)(2x + 3)$ **b** $(3x + 1)(2x + 5)$ **c** $(2x + 1)(x + 3)$

d $(3x - 1)(3x - 4)$ **e** $(5x + 3)(2x + 3)$ **f** $2(3x - 2)(2x - 5)$

5 **a** $\frac{2(x+2)}{x-1}$ **b** $\frac{x}{x-1}$ **c** $\frac{x+2}{x}$

d $\frac{x}{x+5}$ **e** $\frac{x+3}{x}$ **f** $\frac{x}{x-5}$

6 **a** $\frac{3x+4}{x+7}$ **b** $\frac{2x+3}{3x-2}$

c $\frac{2-5x}{2x-3}$ **d** $\frac{3x+1}{x+4}$

7 $(x + 5)$

8 $\frac{4(x+2)}{x-2}$

1.3 Laws of indices

1 **a** 1 **b** 1 **c** 1 **d** 1

2 **a** 7 **b** 4 **c** 5 **d** 2

3 **a** 125 **b** 32 **c** 343 **d** 8

4 **a** $\frac{1}{25}$ **b** $\frac{1}{64}$ **c** $\frac{1}{32}$ **d** $\frac{1}{36}$

5 **a** $\frac{3x^3}{2}$ **b** $5x^2$ **c** $3x$ **d** $\frac{y}{2x^2}$

e $y^{\frac{1}{2}}$ **f** c^{-3} **g** $2x^6$ **h** x

6 **a** $\frac{1}{2}$ **b** $\frac{1}{9}$ **c** $\frac{8}{3}$

d $\frac{1}{4}$ **e** $\frac{4}{3}$ **f** $\frac{16}{9}$

7 **a** x^{-1} **b** x^{-7} **c** $x^{\frac{1}{4}}$

d $x^{\frac{2}{5}}$ **e** $x^{-\frac{1}{3}}$ **f** $x^{-\frac{2}{3}}$

8 **a** $\frac{1}{x^3}$ **b** 1 **c** $\sqrt[5]{x}$

d $\sqrt[5]{x^2}$ **e** $\frac{1}{\sqrt{x}}$ **f** $\frac{1}{\sqrt[4]{x^3}}$

9 **a** $5x^{\frac{1}{2}}$ **b** $2x^{-3}$ **c** $\frac{1}{3}x^{-4}$

d $2x^{-\frac{1}{2}}$ **e** $4x^{-\frac{1}{3}}$ **f** $3x^0$

10 **a** $x^3 + x^{-2}$ **b** $x^3 + x$ **c** $x^{-2} + x^{-7}$

1.4 Surds

1 **a** $3\sqrt{5}$ **b** $5\sqrt{5}$ **c** $4\sqrt{3}$ **d** $5\sqrt{7}$

e $10\sqrt{3}$ **f** $2\sqrt{7}$ **g** $6\sqrt{2}$ **h** $9\sqrt{2}$

2 **a** $15\sqrt{2}$ **b** $\sqrt{5}$ **c** $3\sqrt{2}$ **d** $\sqrt{3}$

e $6\sqrt{7}$ **f** $5\sqrt{3}$

3 **a** -1 **b** $9-\sqrt{3}$ **c** $10\sqrt{5}-7$ **d** $26-4\sqrt{2}$

4 **a** $\frac{\sqrt{5}}{5}$ **b** $\frac{\sqrt{11}}{11}$ **c** $\frac{2\sqrt{7}}{7}$ **d** $\frac{\sqrt{2}}{2}$

e $\sqrt{2}$ **f** $\sqrt{5}$ **g** $\frac{\sqrt{3}}{3}$ **h** $\frac{1}{3}$

5 **a** $\frac{3+\sqrt{5}}{4}$ **b** $\frac{2(4-\sqrt{3})}{13}$ **c** $\frac{6(5+\sqrt{2})}{23}$

6 $x - y$

7 **a** $3+2\sqrt{2}$ **b** $\frac{\sqrt{x}+\sqrt{y}}{x-y}$

2 Solving quadratic equations

2.1 Factorising

1 **a** $x = 0$ or $x = -\frac{2}{3}$ **b** $x = 0$ or $x = \frac{3}{4}$

c $x = -5$ or $x = -2$ **d** $x = 2$ or $x = 3$

e $x = -1$ or $x = 4$ **f** $x = -5$ or $x = 2$

g $x = 4$ or $x = 6$ **h** $x = -6$ or $x = 6$

i $x = -7$ or $x = 4$ **j** $x = 3$

k $x = -\frac{1}{2}$ or $x = 4$ **l** $x = -\frac{2}{3}$ or $x = 5$

2 **a** $x = -2$ or $x = 5$ **b** $x = -1$ or $x = 3$

c $x = -8$ or $x = 3$ **d** $x = -6$ or $x = 7$

e $x = -5$ or $x = 5$ **f** $x = -4$ or $x = 7$

g $x = -3$ or $x = 2\frac{1}{2}$ **h** $x = -\frac{1}{3}$ or $x = 2$

2.2 Using the Formula

1 **a** $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$ **b** $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

2 $x = \frac{7+\sqrt{41}}{2}$ or $x = \frac{7-\sqrt{41}}{2}$

3 $x = \frac{-3+\sqrt{89}}{20}$ or $x = \frac{-3-\sqrt{89}}{20}$

4 **a** $x = \frac{7+\sqrt{17}}{8}$ or $x = \frac{7-\sqrt{17}}{8}$

b $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$

c $x = -1\frac{2}{3}$ or $x = 2$

3 Simultaneous Equations

3.1 Linear - Elimination

1 $x = 1, y = 4$ **2** $x = 3, y = -2$

3 $x = 2, y = -5$ **4** $x = 3, y = -\frac{1}{2}$

5 $x = 6, y = -1$ **6** $x = -2, y = 5$

3.2 Linear - Substitution

1 $x = 9, y = 5$

2 $x = -2, y = -7$

3 $x = \frac{1}{2}, y = 3\frac{1}{2}$

4 $x = \frac{1}{2}, y = 3$

5 $x = -4, y = 5$

6 $x = -2, y = -5$

7 $x = \frac{1}{4}, y = 1\frac{3}{4}$

8 $x = -2, y = 2\frac{1}{2}$

9 $x = -2\frac{1}{2}, y = 5\frac{1}{2}$

3.3 Linear and Quadratic

1 $x = 1, y = 3$
$x = -\frac{9}{5}, y = -\frac{13}{5}$

2 $x = 2, y = 4$
$x = 4, y = 2$

3 $x = 1, y = -2$
$x = 2, y = -1$

4 $x = 4, y = 1$
$x = \frac{16}{5}, y = \frac{13}{5}$

5 $x = 3, y = 4$
$x = 2, y = 1$

6 $x = 7, y = 2$
$x = -1, y = -6$

7 $x = 0, y = 5$
$x = -5, y = 0$

8 $x = -\frac{8}{3}, y = -\frac{19}{3}$
$x = 3, y = 5$

9 $x = -2, y = -4$
$x = 2, y = 4$

10 $x = \frac{5}{2}, y = 6$
$x = 3, y = 5$

11 $x = \frac{1+\sqrt{5}}{2}, y = \frac{-1+\sqrt{5}}{2}$
$x = \frac{1-\sqrt{5}}{2}, y = \frac{-1-\sqrt{5}}{2}$

12 $x = \frac{-1+\sqrt{7}}{2}, y = \frac{3+\sqrt{7}}{2}$
$x = \frac{-1-\sqrt{7}}{2}, y = \frac{3-\sqrt{7}}{2}$

4 Linear inequalities

1 **a** $x > 4$ **b** $x \le 2$ **c** $x \le -1$
d $x > -\frac{7}{2}$ **e** $x \ge 10$ **f** $x < -15$

2 **a** $x < -20$ **b** $x \le 3.5$ **c** $x < 4$

3 **a** $x \le -4$ **b** $-1 \le x < 5$ **c** $x \le 1$
d $x < -3$ **e** $x > 2$ **f** $x \le -6$

4 **a** $t < \frac{5}{2}$ **b** $n \ge \frac{7}{5}$

5 **a** $x < -6$ **b** $x < \frac{3}{2}$

6 $x > 5$ (which also satisfies $x > 3$)

5 Straight line Graphs

5.1 Equations

1 **a** $m = 3, c = 5$ **b** $m = -\frac{1}{2}, c = -7$

c $m = 2, c = -\frac{3}{2}$ **d** $m = -1, c = 5$

e $m = \frac{2}{3}, c = -\frac{7}{3}$ or $-2\frac{1}{3}$ **f** $m = -5, c = 4$

2 **a** $x + 2y + 14 = 0$ **b** $2x - y = 0$ **c** $2x - 3y + 12 = 0$ **d** $6x + 5y + 10 = 0$

3 $y = 4x - 3$

4 $y = -\frac{2}{3}x + 7$

5 **a** $y = 2x - 3$ **b** $y = -\frac{1}{2}x + 6$ **c** $y = 5x - 2$ **d** $y = -3x + 19$

6 $y = -\frac{3}{2}x + 3$, the gradient is $-\frac{3}{2}$ and the y-intercept is 3.
The line intercepts the axes at (0, 3) and (2, 0).
Students may sketch the line or give coordinates that lie on the line such as $\left(1, \frac{3}{2}\right)$ or $(4, -3)$.

5.2 Parallel and Perpendicular

1 **a** $y = 3x - 7$ **b** $y = -2x + 5$ **c** $y = -\frac{1}{2}x$ **d** $y = \frac{3}{2}x + 8$

2 $y = -2x - 7$

3 **a** $y = -\frac{1}{2}x + 2$ **b** $y = 3x + 7$ **c** $y = -4x + 35$ **d** $y = \frac{5}{2}x - 8$

4 **a** $y = -\frac{1}{2}x$ **b** $y = 2x$

5 **a** Parallel **b** Neither **c** Perpendicular
d Perpendicular **e** Neither **f** Parallel

6 **a** $x + 2y - 4 = 0$ **b** $x + 2y + 2 = 0$ **c** $y = 2x$

6 Trigonometry

6.1 Right-angled

1	**a**	6.49 cm	**b**	6.93 cm	**c**	2.80 cm		
	d	74.3 mm	**e**	7.39 cm	**f**	6.07 cm		
2	**a**	36.9°	**b**	57.1°	**c**	47.0°	**d**	38.7°

3 5.71 cm

4 20.4°

5	**a**	45°	**b**	1 cm	**c**	30°	**d**	$\sqrt{3}$ cm

6.2 Cosine Rule

1	**a**	6.46 cm	**b**	9.26 cm	**c**	70.8 mm	**d**	9.70 cm
2	**a**	22.2°	**b**	52.9°	**c**	122.9°	**d**	93.6°
3	**a**	13.7 cm	**b**	76.0°				

6.3 Sine Rule

1	**a**	4.33 cm	**b**	15.0 cm	**c**	45.2 mm	**d**	6.39 cm
2	**a**	42.8°	**b**	52.8°	**c**	53.6°	**d**	28.2°
3	**a**	8.13 cm	**b**	32.3°				

6.4 Area of a triangle

1	**a**	18.1 cm^2	**b**	18.7 cm^2	**c**	693 mm^2

2 5.10 cm

3	**a**	6.29 cm	**b**	84.3°	**c**	5.73 cm	**d**	58.8°

4 15.3 cm

Hegarty Transition clips

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